About the Lesson

In this activity, students will build on their comprehension of functions defined by a definite integral, where the independent variable is an upper limit of integration. Students are led to the brink of a discovery of a discovery of the Fundamental Theorem of Calculus, that $\frac{d}{dx}\int_0^x f(t)dt = f(x)$. As a result, students will:

- Graphical estimate the signed area under the curve in a given interval.
- Algebraically use the Integral to obtain the exact value of a definite integral.

Vocabulary

- signed area
- Fundamental Theorem

Teacher Preparation and Notes

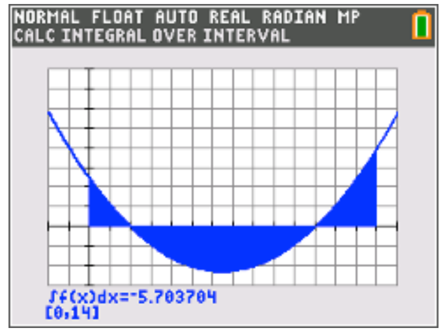
- This investigation should follow coverage of the definition of a definite integral, and the relationship between the integral of a function and the area of a region bounded by the graph of a function and the x-axis.
- Before doing this activity, students should understand that if $a < b$ and $f(x) > 0$, then:
  - $\int_a^b f(x) > 0$
  - $\int_a^b -f(x) < 0$
  - $\int_b^a f(x) < 0$
  - $\int_b^a -f(x) > 0$

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

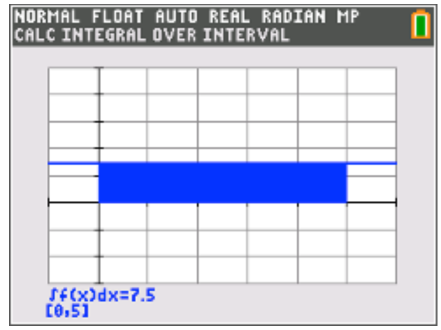
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- FTC_Student.pdf
- FTC_Student.doc

Problem 1 – Constant Integrand

Students explore the function $\int_0^x 1.5dt$. They should notice that there is a constant rate of change in the graph of $f(x) = \int_0^x 1.5dt$. This rate of change is 1.5.



1. Use the **Integrate** command illustrated above to complete the table.

Answers:

x	$\int_0^x 1.5dt$
1	1.5
2	3
3	4.5
4	6
5	7.5

2. If $x = 0$, what is $\int_0^x 1.5dt$? Why?

Answer: $\int_0^0 1.5dt = 0$; This integral function begins accumulating signed areas starting at the lower limit of zero. Because this integral function also stops accumulating signed areas at zero, it has not yet had the chance to accumulate any signed areas. Another other way to view the result of zero is that there is zero area under the graph of $y = 1.5$ from $x = 0$ to $x = 0$.

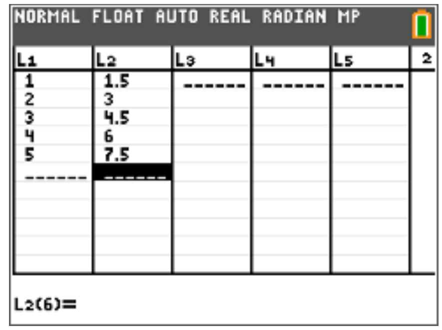
3. For every 1 unit that x changes, how much does $\int_0^x 1.5dt$ change?

Answer: 1.5 units.

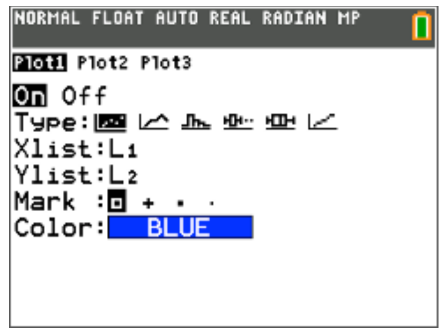
4. If you were to graph the ordered pairs $\left(x, \int_0^x 1.5\,dt\right)$, what would the graph look like?

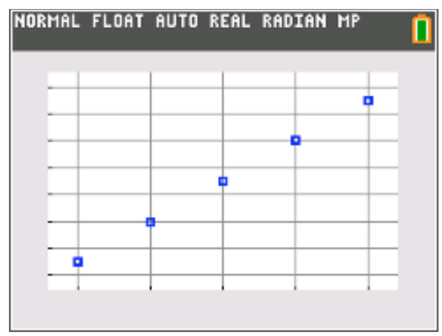
Answer: The graph would be a line through the origin with slope 1.5.

To enter the data into **L_1** and **L_2**, students will need to press [stat] and choose edit.



Once the data is entered, have the students press [2nd] [y=] and select **Plot1**. Then, have the students match the screen to the right, press [zoom] and select **ZoomStat** to view the scatter plot.





5. What does your graph look like? Was this graph what you predicted in Question 4?

<u>Sample Answer:</u> A line; yes

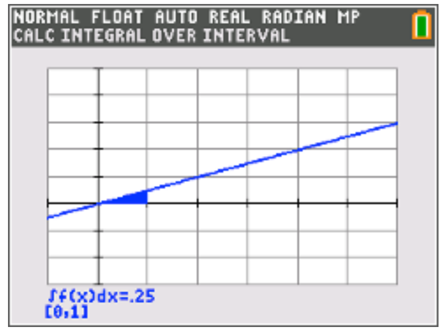
6. If you changed the integrand from 1.5 to 0.5, what would the graph of $\left(x, \int_0^x 0.5\,dt\right)$ look like?

<u>Answer:</u> The same as before, except the slope would be 0.5 instead of 1.5.

Problem 2 – Non-Constant Integrand

Students will investigate the behavior of $f(x) = \int_0^x \frac{t}{2}dt$.

Students should note that this function changes at a non-constant rate



7. Complete the table.

Answers:

x	$\int_0^x \frac{t}{2}dt$
1	0.25
2	1
3	2.25
4	4
5	6.25

8. If $x = 0$, what is $\int_0^x \frac{t}{2}dt$? Why?

Answer: $\int_0^0 \frac{t}{2}dt = 0$; The height and the length of the triangle are 0 so the area is 0.

9. Explain why, when x increases by 1, the value of $\int_0^x \frac{t}{2}dt$ does not increase by the same amount every time.

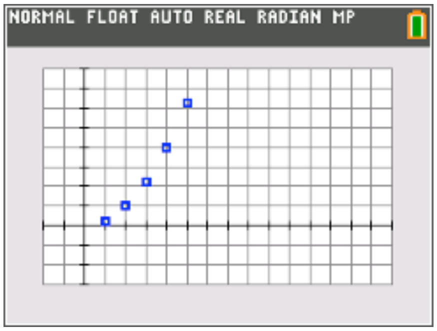
Answer: The area changes by a different amount each time because both the height and width are increasing.

10. **a.** Is the graph of $\left(x, \int_0^x \frac{t}{2}\,dt\right)$ linear? Explain.

Answer: The graph is not linear; *y* values are increasing at an increasing rate.

b. Using the values in the table in Question 7, enter the data into lists **L1** and **L2**. Then plot the data.

Answer:

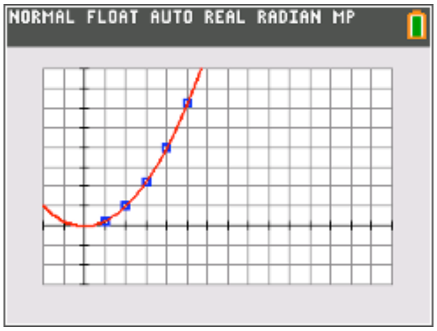


c. Describe the shape of the graph.

Answer: It looks like a parabola (quadratic).

d. By using guess 'n check, try to find the equation that models this data and graph it.

Answer: $y = \frac{1}{4}x^2$ seems to model the data.

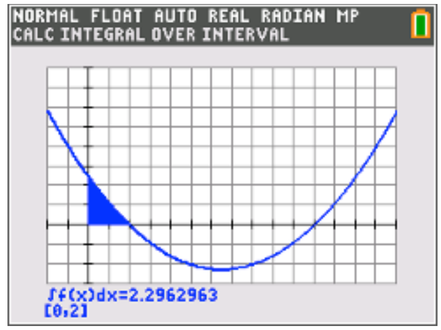


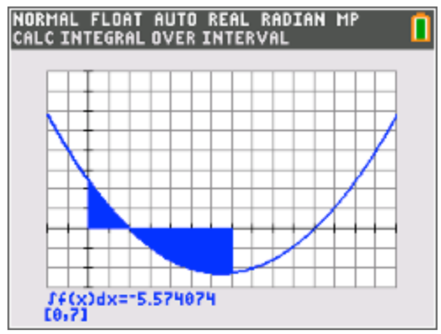
Problem 3 – An Integrand that Changes Sign

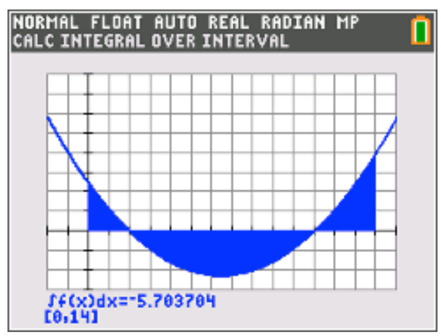
11. Complete the table.

Answers:

x	$\int_0^x \frac{t^2 - 13t + 22}{9} dt$
1	$\frac{95}{54} \approx 1.76$
2	$\frac{62}{27} \approx 2.30$
3	$\frac{11}{6} \approx 1.83$
4	$\frac{16}{27} \approx 0.59$
5	$-\frac{65}{54} \approx -1.20$
6	$-\frac{10}{3} \approx -3.33$
7	$-\frac{301}{54} \approx -5.57$
8	$-\frac{208}{27} \approx -7.70$
9	$-\frac{19}{2} = -9.50$
10	$-\frac{290}{27} \approx -10.74$
11	$-\frac{605}{54} \approx -11.20$
12	$-\frac{32}{3} \approx -10.67$
13	$-\frac{481}{54} \approx -8.91$
14	$-\frac{154}{27} \approx -5.70$







12. At what value of *x* does the integral's value begin to decrease?

Answer: After $x = 2$, the integral value begins to decrease.

13. a. What are all the values of *x* for which the definite integral's value is decreasing?

Answer: The values for *x* in which the integral decreases are $2 < x < 11$.

b. What is true at these values of *x*?

Answer: The function is negative.

14. a. What are all the values of *x* for which the integral's value is increasing?

Answer: The values for which the integral is increasing are $x < 2$, $x > 11$

b. What is true of the integrand at these values of *x*?

Answer: The function is positive.

15. a. What is the smallest value of the integral, and at what value of *x* is this reached?

Answer: The table seems to indicate $x = 11$.

b. What happens with the integrand at this value of *x*?

Answer: The integrand is negative right before x = 11 and positive after x = 11.

16. Is the connection between the location of the minimum value of $\int_0^x \frac{t^2 - 13t + 22}{9} dt$ and the sign change of the integrand from negative to positive one you that you have seen before? If so, in what context?

Answer: Yes, we have seen a similar situation. The minimum occurs on a function where the function stops decreasing and starts increasing.