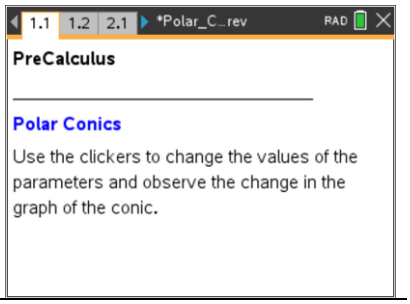
Polar Conics
Student Activity

Name _____

Class _____

Open the TI-Nspire document *Polar_Conics.tns.*

In this activity, you will explore how to express an ellipse, a hyperbola, and a parabola from a single equation and investigate the different parameters of the equation.



A conic is defined as the locus of points in a plane whose distance from a fixed point (focus) and a fixed line (directrix) is a constant ratio. This ratio is called the eccentricity, *e*, of the conic. The polar notation for the ellipse, hyperbola, and parabola is given by the equation:

$$r = \frac{ed}{1 \pm e\cos(\theta)} \quad \textbf{OR} \quad r = \frac{ed}{1 \pm e\sin(\theta)}$$

where *e* is the eccentricity and *d* is the distance from the origin to the directrix.

By expressing the equation in polar coordinates, we can generate all three types of conics from a single equation.

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Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. Use the clicker to change the values of the eccentricity, *e*. For what values of *e* is the conic a parabola? An ellipse? A hyperbola?

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2. Use the clicker to change the values of *d*, the distance between a point on the conic and the directrix.
   a. Set *e* = 1. When the conic is a parabola, what effect does *d* have on the graph of the function?

   b. Set *e* < 1. When the conic is an ellipse, what effect does *d* have on the graph of the function?

   c. When the conic is a hyperbola, what effect does *d* have on the graph of the function ?

3. Adjust the parameters to create an ellipse that is 9 units in width, and make a note of those parameters. Are these the only parameters that will create such an ellipse? Explain.

4. Adjust the parameters to create a hyperbola for which the vertices of the branches are 6 units apart, and make a note of those parameters. Are these the only parameters that will create such a hyperbola? Explain.

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5. Use the clicker to adjust the value of *a*, the phase shift.
   a. Set *e* = 1. When the conic is a parabola, what effect does *a* have on the graph of the function?

   b. Set *e* < 1. When the conic is an ellipse, what effect does *a* have on the graph of the function?

   c. Set *e* > 1. When the conic is a hyperbola, what effect does *a* have on the graph of the function?

6. Is it possible to adjust the values of *a* and *e* so that the resulting conic is a parabola centered about the y-axis? If so, what parameters yield this result? If not, explain why not.

7. Which type of conic will result from each of the following equations? How do you know?
   a. $r = \frac{10}{1+3\cos(\theta-5)}$
   b. $r = \frac{3}{1-\cos(\theta-6)}$
   c. $r = \frac{20}{1-0.5\cos(\theta-2)}$