Math Objectives

- Use a table and a graph to compare the changes in linear and exponential expressions as x is increased
- Recognize that as x increases a linear expression increases at a constant rate (additively) while an **exponential function** increases multiplicatively
- Recognize that an exponential function with a base greater than 1 will never be less than or equal to zero, but will get smaller and smaller as x decreases
- Determine whether a graph represents a linear or exponential function
- Use appropriate tools strategically (CCSS Mathematical Practice)
- Construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice)

Vocabulary

- exponential function

About the Lesson

- Students will move a point that changes the value of x and observe and compare the values of a linear expression and an exponential expression.

Related Lessons

- After this lesson: Domain and Range of Exponential Functions

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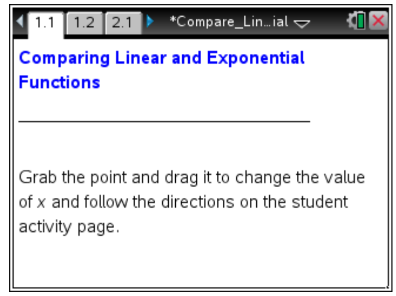
- Using Class Capture to compare linear and exponential expressions.
- Students will compare and contrast linear and exponential functions using a Notes page and Class Capture.
- Use the Teacher Software or Live Presenter to review student documents and discuss examples as a class.
- Use Quick Poll to assess student understanding throughout the lesson.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Files:

Student Activity

- Compare_Linear_Exponential_Student.pdf
- Compare_Linear_Exponential_Student.doc

TI-Nspire document

- Compare_Linear_Exponential.tns

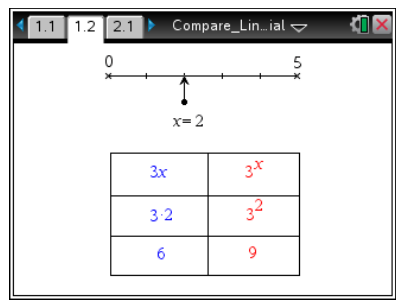
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word point appears. Then press ctrl to grab the point and close the hand ( ).

Teacher Note: This lesson can be used to probe more deeply into the behavior of exponential functions by changing the base in the .tns document, using numbers such as 2 or 0.5 for the base.

Move to page 1.2.



1. Complete the table below by moving the point. Which column is growing faster?

   <u>Answer:</u> The 3^x column is growing faster.

x	$3x$	3^x
0	0	1
1	3	3
2	6	9
3	9	27
4	12	81
5	15	243

2. a. As x increases from 2 to 3 in the table, how does the value of $3x$ change?

   <u>Answer:</u> The value of $3x$ increases by 3.

b. As x increases by 1, describe the pattern you notice in the numbers in the $3x$ column of the table.

Answer: The numbers increase by 3 each time.

> **Teacher Tip:** Check for student understanding of the repeated operation of the addition of 3 at this point.

3. a. As x increases from 2 to 3 in the table, how does the value of 3^x change?

Answer: It triples; it increases 3 times as much.

b. As x increases from 3 to 4 in the table, how does the value of 3^x change?

Answer: It triples; it increases 3 times as much.

c. As x increases by 1, describe the pattern you notice in the numbers in the 3^x column of the table.

Answer: The numbers are always being multiplied by 3. The values triple.

> **Teacher Tip:** Since the rate of change for $3x$ is constant, students might initially examine the values of 3^x in terms of rate of change. For instance, a student could respond "the value of 3^x increases by 18." In this case, you might ask the student if this pattern holds true for all changes in the value of 3^x. Since it does not, encourage the student to search for another pattern in the table.

4. Complete the bottom row of the table for $x = 6$. How did you determine the values for $3x$ and 3^x?

Answer: Students might say that they added 3 to 15 (previous row) to get 18 and multiplied 243 by 3 to get 729; or any other acceptable method.

x	$3x$	3^x
6	18	729

5. Why are the values for 3^x increasing faster than the values for $3x$?

Answer: The values of 3^x are increasing faster than $3x$ because you multiply the previous number by 3 instead of adding 3 to the previous number. When you have a whole number greater than 1 repeatedly multiplied by 3, the result gets larger faster than when you repeatedly add 3.

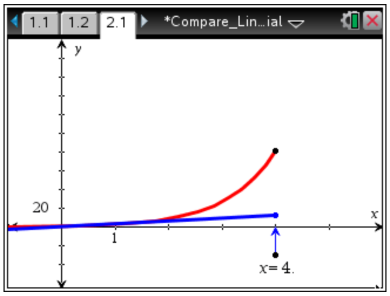
For example, if the whole number were 2, $2 \cdot 3 = 6$ while $2 + 3 = 5$. The product is larger at the beginning, and the sum will never catch up. $2 \cdot 3 \cdot 3 = 18$ while $2 + 3 + 3 = 8$.

> **Teacher Tip:** While multiplying whole numbers greater than 1 by a positive integer greater than 1 makes the product increase, students should recognize that when a fraction between 0 and 1 is multiplied by a constant multiplier greater than one, the results get smaller and smaller.
> Ex: 1/3, 1/9, 1/27,…
> You might want to have students reflect on how multiplication works as repeated addition, that is $3 \cdot 2$ means two 3s or $3 + 3$. Thus, comparing 3^x to $3x$ going from $x = 5$ to $x = 6$ means for $3x$ you have five 3s or $3 + 3 + 3 + 3 + 3$ and the next term would have six 3s or $(3 + 3 + 3 + 3 + 3) + 3$ where you added a 3. With 3^5, the next term would be found by multiplying three sets of 3^5 by 3. Or, $3 \cdot 3^5 = (3^5 + 3^5 + 3^5)$. Two 3^5s were actually added to the previous term

6. The function $f(x) = 3^x$ is called an ***exponential function***, while the function $f(x) = 3x$ is a ***linear function***. Describe the difference in the two functions.

Answer: A linear function has the variable as a factor in defining the function. In an exponential function, the variable is part of the exponent.

Move to page 2.1.



7. Drag the point on the arrow to the right to produce two graphs—one red and one blue. Use the information from the table in question 1 to identify which graph represents an exponential function and which graph represents a linear function. Justify your answer.

Answer: The dashed graph remains closer to the x-axis and is $f(x) = 3x$ because it is increasing at a slower rate than the graph $f(x) = 3^x$. $f(x) = 3x$ increases at a constant rate, 3 units vertically for every 1 unit horizontally. The solid graph, $f(x) = 3^x$, increases at an increasing rate.

8. How do the graphs of $f(x) = 3x$ and $f(x) = 3^x$ support your response to question 5?

Answer: When comparing the *y*-values, for $f(x)=3x$, each time *x* increases by 1 unit, the *y*-value increases by 3 units. For $f(x) = 3^x$, each time *x* increases by 1 unit, the new *y*-value is 3 times the previous *y*-value.

9. Aaron says that the values of $f(x) = 5^x$ will increase faster than the values of the linear function $f(x) = 5x$. Do you agree or disagree? Support your answer.

Answer: I agree with Aaron, because for $f(x) = 5^x$, the *y*-values be multiplied by 5 every time the x-value is increased by 1. For $f(x) =5x$, 5 will be added to the previous *y*-value each time the *x*-value increases by 1.

Teacher Tip: This might be a good time to ask students to give you examples of other linear or exponential functions.

TI-Nspire Navigator Opportunity: *Quick Poll and Class Capture*
See Notes 1 and 2 at the end of this lesson.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- Expressions of the form $3x$ increase by repeated addition.
- Expressions of the form 3^x increase by repeated multiplication.
- Graphs of linear functions increase at a constant rate.
- Graphs of exponential functions of the form $y = b^x$ where *b* is greater than 1 increase faster than graphs of linear functions of the form $y = bx$.
- Exponential functions of the form $y = b^x$ where *b* is greater than 0 will never have values for $f(x)$ that are 0 or negative.

TI-Nspire Navigator Opportunity: *Quick Poll and Class Capture*
See Note 3 at the end of this lesson.

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Note 1

Question 9, *Quick Poll* and *Class Capture*

Use Quick Poll to determine the number of students agreeing with the statement in Question 9.

Have student press [ctrl] [G] to show the function entry line on page 2.1. Then, press the ▲ on the Nav Pad twice to move to f1(x) and press the ◀ until the cursor is between the base and the exponent. Press [del] and change the base from 3 to 5. Press [enter].

Have student press [ctrl] [G] again and press the ▲ on the Nav Pad once to move to f2(x). Move the cursor until it is to the right of 3 and press [del]. Change the 3 to a five. Press [enter].

Students then drag the point on the arrow to the right to see the 2 graphs. Use Class capture to view the screens. Was Aaron correct?

You might want to have different groups of students change the coefficient of the linear equation and the base on the exponential equation to other numbers greater than one and use Class Capture to compare the results. Numbers between 0 and 1 can be used, but you will need to press **Menu > Window** > **Zoom Out**, and [enter] before moving the point on the arrow to the **left.**

Note 2

Question 9, *Quick Poll*

1. Have students enter a linear function.
2. Have students enter an exponential function.

Note 3

Question Wrap Up, *Class Capture*

Have student press [ctrl] + [I] and choose *Notes* page to add a new page to the file. Have students compare and contrast linear and exponential function on the page. Capture their screens and discuss their responses.