



MATH AND SCIENCE @ WORK

AP* CALCULUS Educator Edition



SPACE SHUTTLE ASCENT

Instructional Objectives

Students will

- create and analyze scatterplots given a table;
- find regression functions using a graphing calculator;
- apply the first derivative test to find local extrema;
- find inflection points to analyze the concavity of a function; and
- make connections to real world problems.

Degree of Difficulty

This problem is very calculator intensive. The students will need to recall multiple calculator functions. Students will also be connecting data and making multiple conjectures relating graphs and tables to a real life situation.

- For the average AP Calculus AB student the problem may be moderate to very difficult depending on calculator experience.
- For the average AP Calculus BC student the problem may be moderately difficult.

Media Resources

This problem has two media files for use during the introduction and post discussion of the problem. These files will engage students in a space shuttle launch and introduce them to the different events that take place during the space shuttle's ascent into space.

- Space Shuttle Launch Video (10 minutes).
This video shows the launch of Space Shuttle Discovery STS 121 mission on July 4, 2006. To access the video, follow the link provided and choose the video titled: "The Rocket's Red Glare." This should be shown to students during the introduction of the problem and is necessary for them to answer part of the problem. It is highly recommended that the instructor become familiar with the video before showing it to students.

http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html

Appendix A provides a discussion guide that can be used with the video.

Grade Level
11-12

Key Topic
Applications of
Differentiation

Degree of Difficulty
Calculus AB:
Moderate to Difficult
Calculus BC: Moderate

Teacher Prep Time
10 minutes

Problem Duration
60-90 minutes

Technology
Graphing Calculator
(TI-83 Family, TI-84 Family)

Materials
Student Edition including:
- Background handout
- Problem worksheet

NCTM Principles and Standards
- Algebra
- Data Analysis and Probability
- Problem Solving
- Communication

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- Google Earth Space Shuttle Ascent Trajectory (5-10 minutes). The file, AscentTrajectory_Tour.kmz, shows the ascent phase of the Space Shuttle Discovery during STS 119 in Google Earth. The file is provided for download with this problem.

Appendix B provides instructions for using this file.

Background

This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the International Space Station, or ISS. However, by 2010 NASA plans to retire the space shuttle in favor of a new Crew Exploration Vehicle, or CEV. Until then, space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.



Figure 1: Space Shuttle Discovery at lift-off during STS 121.



Figure 2: Onlookers view the launch of STS 121.

The ascent phase begins at liftoff and ends at insertion into a circular or elliptical orbit around the Earth. To reach the minimum altitude required to orbit the Earth, the space shuttle must accelerate from zero to 8,000 meters per second (almost 18,000 miles per hour) in eight and a half minutes. It takes a very unique vehicle to accomplish this.

There are three main components of the space shuttle that enable the launch into orbit. The orbiter is the main component. It not only serves as the crew's home in space and is equipped to dock with the International Space Station, but contains maneuvering engines for finalizing orbit. The external tank, the largest component of the space shuttle, supplies the propellant to the orbiter's three main maneuvering engines. The two solid rocket boosters, the third component, provide the main thrust at launch and are attached to the sides of the external tank (Figure 1). The components of the space shuttle experience changes in position, velocity and acceleration during the ascent into space. These changes can be seen when one takes a closer look at the entire ascent process (Figure 3).



The ascent process begins with the liftoff from the launch pad. Propellant is being burned from the Solid Rocket Boosters, or SRB, and the external tank, or ET, causing the space shuttle to accelerate very quickly. This high-rate of acceleration as the space shuttle launches through the Earth's atmosphere causes a rapid increase in dynamic pressure, known as Q in aeronautics. The structure of the space shuttle can only withstand a certain level of dynamic pressure (critical Q) before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 70%. At about one minute after launch the dynamic pressure reaches its maximum level (max Q). The air density then drops rapidly due to the thinning atmosphere and the space shuttle can be throttled to full power without fear of structural damage.

At about 2 minutes after launch, the atmosphere is so thin that the dynamic pressure drops down to zero. The SRB, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the external tank. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second.

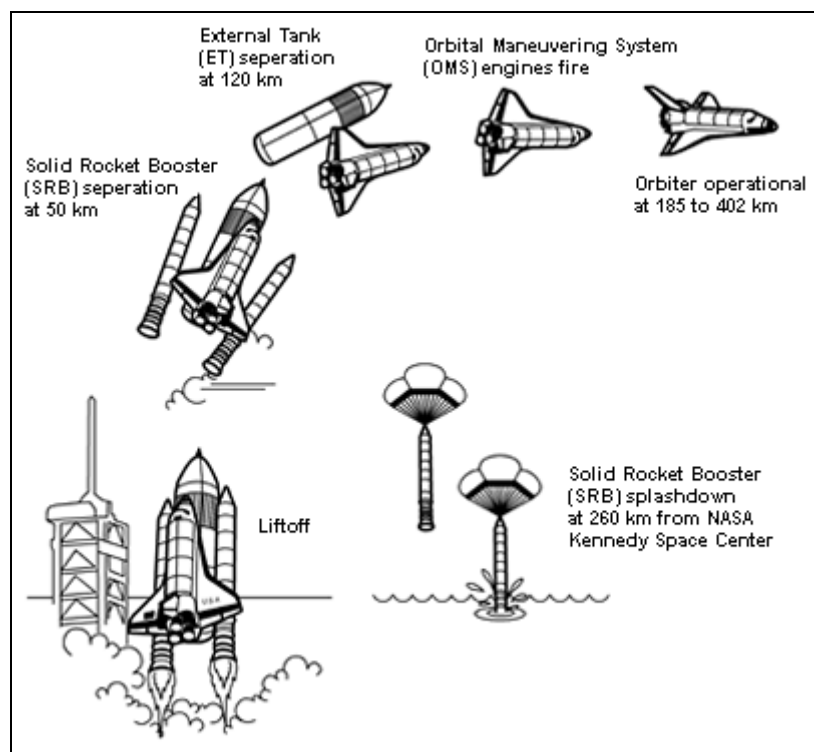


Figure 3: Space Shuttle Ascent Process

The second stage of ascent lasts about six and a half minutes. The space shuttle gains more altitude above Earth and the speed increases to the nearly 7,850 m/s (17,500 mph) required to achieve orbit. The main engines are commanded by the onboard computer to reduce power, ensuring that acceleration of the space shuttle does not exceed 29.4 m/s^2 (3 g). Within thirty seconds the space shuttle reaches Main Engine Cut-Off, or MECO. For the next eleven seconds, the space shuttle coasts through space. At nine minutes, the command to jettison the nearly empty external tank is given by the onboard computer leaving the space shuttle's maneuvering engines to control any movement needed to achieve final orbit.

As we look to the future of space exploration, the ascent stage will remain a critical part of any successful mission.



AP Course Topics

Functions, Graphs, and Limits

- Analysis of Graphs
 - With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function

Derivatives

- Concept of the derivative
 - Derivative presented graphically, numerically, and analytically
- Derivative as a function
 - Corresponding characteristics of graphs of f and f'
 - Relationship between the increasing and decreasing behavior of f and the sign of f'
- Second derivatives
 - Corresponding characteristics of the graphs of f , f' , and f''
 - Relationship between the concavity of f and the sign of f''
 - Points of inflection as places where concavity changes
- Application of derivatives
 - Analysis of curves, including the notions of monotonicity and concavity
 - Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

NCTM Principles and Standards

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.

Data Analysis and Probability

- Select and use appropriate statistical methods to analyze data.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Monitor and reflect on the process of mathematical problem solving.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.



Problem

Before each mission, projected data is compiled to assist in the launch of the space shuttle to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as the weight of the space shuttle, propellant used, mass of payload being carried to space, and mass of payload returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple scenarios through computer modeling, information is compiled in a table showing exactly what should happen each second of the ascent. This information helps the NASA engineers and flight controllers to quickly evaluate whether or not the space shuttle is operating as expected during the launch phase.

Table 1: STS-121 Ascent Data

Time (s)	Altitude (m)	Velocity (m/s)	Acceleration (m/s ²)
0	-8	0	2.45
20	1244	139	18.62
40	5377	298	16.37
60	11,617	433	19.40
80	19,872	685	24.50
100	31,412	1026	24.01
120	44,726	1279	8.72
140	57,396	1373	9.70
160	67,893	1490	10.19
180	77,485	1634	10.68
200	85,662	1800	11.17
220	92,481	1986	11.86
240	98,004	2191	12.45
260	102,301	2417	13.23
280	105,321	2651	13.92
300	107,449	2915	14.90
320	108,619	3203	15.97
340	108,942	3516	17.15
360	108,543	3860	18.62
380	107,690	4216	20.29
400	106,539	4630	22.34
420	105,142	5092	24.89
440	103,775	5612	28.03
460	102,807	6184	29.01
480	102,552	6760	29.30
500	103,297	7327	29.01
520	105,069	7581	0.10

Note: Notice from the table that the altitude is negative at liftoff. Zero altitude can be described as a specific distance from the center of the Earth. Since the Earth is not perfectly spherical the location of the launch just happens to be below this specified point. Also, because this is a calculated number, some degree of error may be present.



On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the ISS. Data for mission STS-121 showing the altitude, velocity, and acceleration of the space shuttle every 20 seconds from liftoff to MECO, is displayed in Table 1.

In the following questions you will use the data from Table 1 to determine a position function, a velocity function and an acceleration function. *Position* is defined as a location in a coordinate system. In this problem, the altitude of the space shuttle in relation to time after launch is the space shuttle's vertical position. *Velocity* is defined as the rate of change of the vertical position. It describes how fast the space shuttle is moving. It is measured in meters per second (m/s). *Acceleration* is defined as the rate of change of velocity. It describes how the velocity of the space shuttle is changing and is measured in meters/second² (m/s²).

Because of the power and speed of lift off, precision is of utmost importance. Keep this in mind as you work with the data in this problem.

Note: To save time and errors the data files for the table to be uploaded to a TI calculator with instructions have been provided on the website.

- A. Enter the data from Table 1 into your graphing calculator. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window. Use the regression functions on your calculator to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.).

Note: When using the regression functions it is sometimes necessary to specify which lists you are using and is very useful to specify where you want the equation placed. For example the command $QUADREG(L1, L2, Y_1)$ will give the quadratic regression equation using L1 as the independent variable and L2 as the dependent variable and will automatically place this equation in Y_1 . You will find the Y variables by typing the VARS key, selecting Y-VARS, and then FUNCTION.

- B. Write the function describing the space shuttle's altitude? Let h stand for altitude and t stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your calculator, keep the original equation in Y_1 to be use for following questions.)
- C. Using your graphing calculator and the NDERIV command, set Y_2 , the altitude function, to be the first derivative and apply the first derivative test to Y_2 , to determine the relative extrema on the interval $[0, 520]$. Remember to adjust your viewing window. Compare the extrema with the data in table 1. How closely does your function represent the data in table 1? What do the extrema represent about the altitude of the space shuttle?
- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in Y_3 . Describe the concavity of the altitude function.
- E. Analyze the altitude function along the interval $[0, 160]$. What is happening to the space shuttle when the concavity changes?
- F. Use your graphing calculator to make a scatterplot of the velocity versus time from Table 1. Explain what is happening to the velocity of the space shuttle over the interval $[0, 520]$. Is there one function that best fits the data? Why or why not? Be careful to select the correct lists when



finding the regressions. What is your velocity function? Let v stand for velocity and t stand for time. Keep this equation in Y_4 on your graphing calculator.

- G. Create a scatterplot of acceleration vs. time. What happens to the space shuttle's acceleration between $[0, 100]$? Why does the space shuttle's acceleration increase in a quadratic fashion along the interval $[120, 460]$? What is happening to the acceleration on the interval $[480, 500]$ and why? Why does the acceleration drop drastically along the interval $[500, 520]$? What happens to the space shuttle during these time intervals?
- H. Is there one regression function that best fits the acceleration data? Why or why not? What is the acceleration function that describes the interval $[120, 460]$? Let a stand for acceleration and t stand for time. Express the coefficients as three significant figures. Keep this equation in Y_5 on your graphing calculator.
- I. Compare the graphs in Y_2 (the first derivative of the altitude function) and in Y_4 (the velocity function found by doing the regression of the data). Do this by turning all other equations off. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.
- J. Now compare the graphs in Y_3 (the second derivative of the altitude function) and in Y_5 (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.

Solution Key (One Approach)

- A. Enter the data from Table 1 into your graphing calculator. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window. Use the regression functions on your calculator to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.).

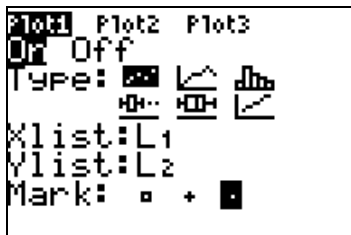
Note: When using the regression functions it is sometimes necessary to specify which lists you are using and is very useful to specify where you want the equation placed. For example the command $QUADREG(L1, L2, Y_1)$ will give the quadratic regression equation using $L1$ as the independent variable and $L2$ as the dependent variable and will automatically place this equation in Y_1 . You will find the Y variables by typing the $VARS$ key, selecting $Y-VARS$, and then $FUNCTION$.

A Quartic equation best fits the data.

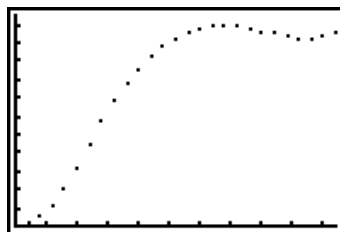
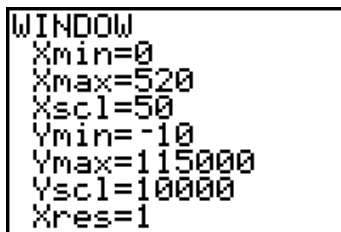
Students will arrive at this conclusion using the following procedures on the graphing calculator.

Step 1: If students do not already have the table values in their calculator, they should go the $STAT$ menu, select $EDIT$, and enter time in $L1$, altitude in $L2$, velocity in $L3$, and acceleration in $L4$.

Step 2: Go to $STAT PLOT$ ($2ND, Y=$); go into $PLOT 1$ and change settings as shown below.

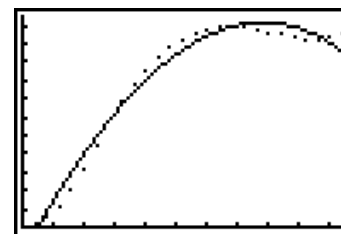
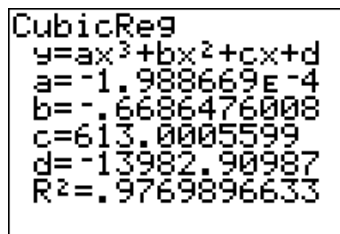
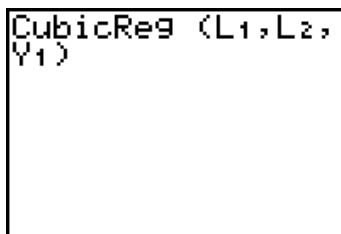


Step 3: Change the window settings to fit the table data and graph. An example is shown.



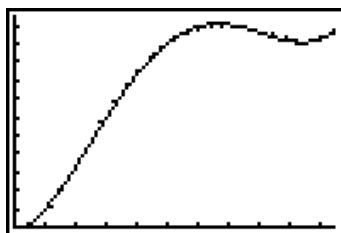
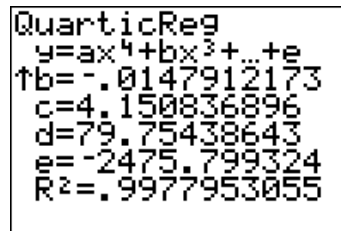
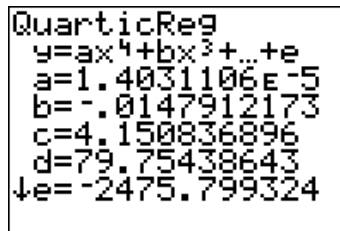
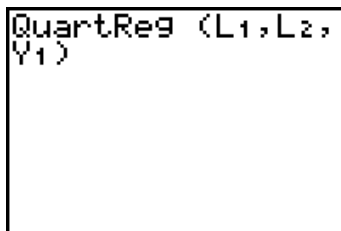
Step 4: Students should see that the data is not very linear and that it is not shaped like a parabola, ruling out linear and quadratic functions. They can do the regressions for cubic and quartic then graph the equations to see which matches the graph better. When they try to do the regressions for exponential and logarithmic, they will get a domain error on their calculator ruling them out.

Cubic Regressions gives the following:



On visual inspection, students will see that the first inflection point is missing and can thus rule out cubic.

Quartic regression gives the following:





The students should see the quartic regression matches the data.

Note: You may want to discuss the R^2 value shown in the regression and what that means about the data. To show this in the regression the diagnostics need to be turned on. This is found by accessing the CATALOG (2ND, 0), scrolling down to DIAGNOSTIC ON, and pressing ENTER.

- B. Write the function describing the space shuttle’s altitude? Let h stand for altitude and t stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your calculator, keep the original equation in Y_1 to be use for following questions.)

Rounding to three decimal places would not be appropriate in this case because that would round the coefficient of x^4 to 0 making the function cubic rather than quartic. If you round the coefficients you would need to round to at least 6 decimal places in order for the function to fit the data well. Using three significant figures would also be appropriate and would still fit the data.

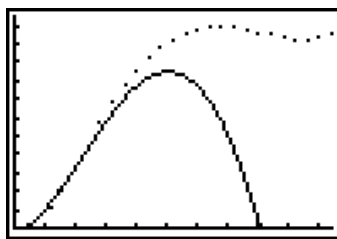
Rounding coefficients to six decimal places gives:

$$h(t) = 0.000014t^4 - 0.014791t^3 + 4.150837t^2 + 79.754386t - 2475.799324$$

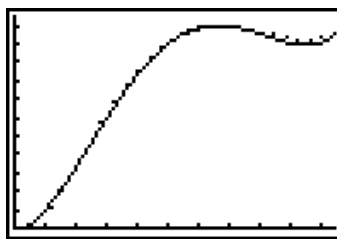
Expressing coefficients in three significant figures gives:

$$h(t) = (1.40 \times 10^{-5})t^4 - (1.48 \times 10^{-2})t^3 + 4.15t^2 + 79.8t - 2480$$

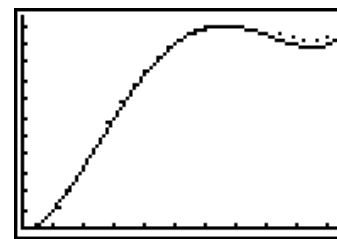
Note: This problem would be good to discuss as a class and come to a consensus on how to represent the equations as a class. Some students will be more familiar with using significant figures than others depending on which Science classes they have taken. This would be a great time to discuss why significant figures are the rule in science rather than rounding. The following screen shots show what students will find as they explore which method to use.



Coefficients rounded to 5 decimal places



Coefficients rounded 6 decimal places



Coefficients expressed as 3 significant figures

- C. Using your graphing calculator and the NDERIV command, set Y_2 , the altitude function, to be the first derivative and apply the first derivative test to Y_2 , to determine the relative extrema on the interval $[0, 520]$. Remember to adjust your viewing window. Compare the extrema with the data in table 1. How closely does your function represent the data in table 1? What do the extrema represent about the altitude of the space shuttle?

The local extrema can be found by finding the zeros of the first derivative of the function.

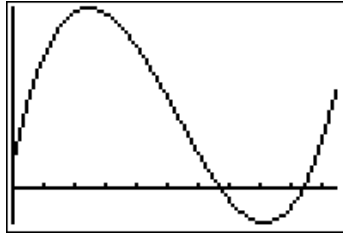
Step 1: Enter the derivative of Y_1 into Y_2 as shown. You will find NDERIV under the MATH key as option 8. You can find Y_1 by pressing the VARS key, selecting Y-VARS, and then FUNCTION.



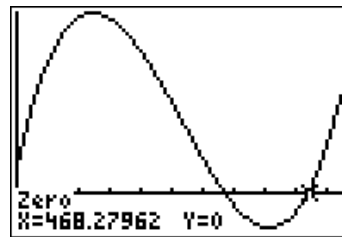
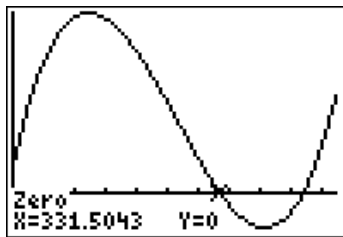
```

Plot1 Plot2 Plot3
Y2=InDeriv(Y1,X,
X)
Y3=
Y4=
Y5=
Y6=
Y7=
    
```

Step 2: Turn off the Y_1 graph and Plot1 and change to an appropriate window by going to option 0: ZOOMFIT under the ZOOM menu.



Step 3: Find the zeros by going to the CALC menu and selecting option 2.



The local extrema are at $t = 332$ seconds and $t = 468$ seconds

By looking at the altitude function one can see when time reaches 332 seconds we have a local maximum and when time reaches 468 seconds we have a local minimum.

On our function when time is 332 seconds the altitude is 110,718 meters. When time is 468 seconds, the altitude is 100,928 meters. Looking at the table, the highest altitude the space shuttle reaches is 108,942 meters and occurs at 340 seconds. The space shuttle then dips down a bit to an altitude of 102,552 meters at 480 seconds before again increasing altitude. So our function is pretty closely aligned with the data in the table.

Note: This calculation may take a moment to load depending on the calculators being used and how much memory is available.

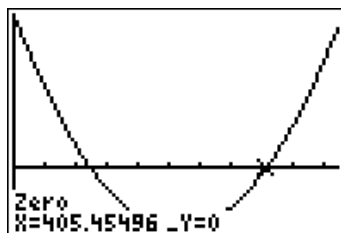
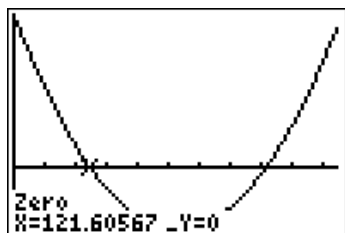
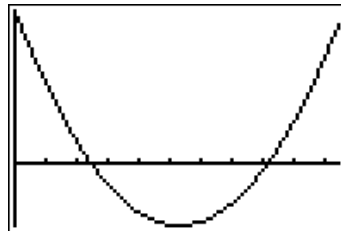
- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in Y_3 . Describe the concavity of the altitude function.

Find where the second derivative of the function equals zero. To do this on the graphing calculator enter the derivative function into Y_3 , turn off Y_1 and Y_2 , change the window (ZOOMFIT), and use the CALC menu to find the zeros.



```

Plot1 Plot2 Plot3
Y2=nDeriv(Y1,X,
)
Y3=2nDeriv(Y2,X,
)
Y4=
Y5=
Y6=
    
```



The points of inflection can be found at $t = 122$ seconds and $t = 405$ seconds

The altitude function is concave up on the interval $[0, 122]$ concave down on the interval $[122, 405]$ and concave up on the interval $[405, 520]$.

Note: This calculation may take a moment to load depending on the calculators being used and how much memory is available.

- E. Analyze the altitude function along the interval $[0, 160]$. What is happening to the space shuttle when the concavity changes?

At approximately 2 minutes (120 seconds) the Solid Rocket Boosters, or SRB, separate from the rocket.

- F. Use your graphing calculator to make a scatterplot of the velocity versus time from Table 1. Explain what is happening to the velocity of the space shuttle over the interval $[0, 520]$. Is there one function that best fits the data? Why or why not? Be careful to select the correct lists when finding the regressions. What is your velocity function? Let v stand for velocity and t stand for time. Keep this equation in Y_4 on your graphing calculator.

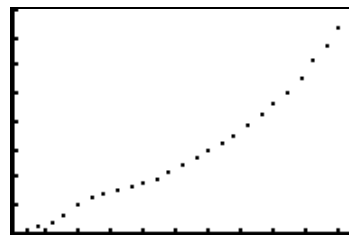
Step 1: On the graphing calculator turn off all equations in $Y=$. Then set plot 2 as shown. Adjust the window and graph.

```

Plot1 Plot2 Plot3
Off Off
Type: [ ] [ ] [ ]
Xlist:L1
Ylist:L3
Mark: [ ] [ ] [ ]
    
```

```

WINDOW
Xmin=0
Xmax=520
Xscl=50
Ymin=0
Ymax=8000
Yscl=1000
Xres=1
    
```

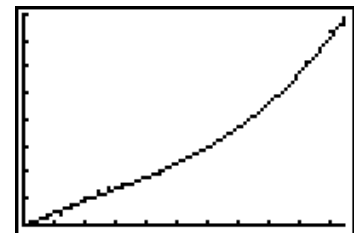
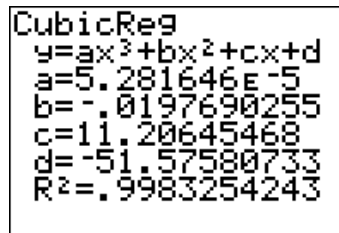
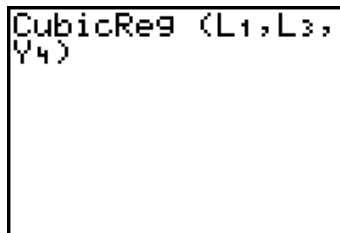


On the interval $[0, 120]$ the velocity is increasing at a fluctuating rate. It continues to increase in an almost quadratic manner until it gets to 500 seconds. Then the velocity appears to stay constant.

Since we used a quartic equation for the altitude function, the velocity function should be a cubic function because it should be the derivative of the altitude function.



Step 2: Calculate the cubic regression equation and place it in Y_4 . Check graph to be sure it matches data.



Rounding coefficients to six decimal places gives:

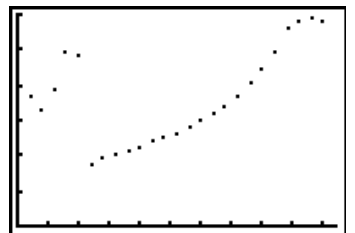
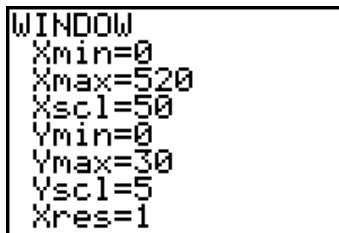
$$v(t) = 0.000053t^3 - 0.019769t^2 + 11.206455t - 51.575807$$

Expressing coefficients in three significant figures gives:

$$v(t) = (5.28 \times 10^{-5})t^3 - (1.98 \times 10^{-2})t^2 + 11.2t - 51.6$$

- G. Create a scatterplot of acceleration vs. time. What happens to the space shuttle's acceleration between $[0, 100]$? Why does the space shuttle's acceleration increase in a quadratic fashion along the interval $[120, 460]$? What is happening to the acceleration on the interval $[480, 500]$ and why? Why does the acceleration drop drastically along the interval $[500, 520]$? What happens to the space shuttle during these time intervals?

On the graphing calculator turn off plot 2 and all equations in $Y=$. Then set plot 3 as shown. Adjust the window and graph.



On the interval $[0, 100]$ the acceleration starts out fairly large, appears to dip and then increases again. This is while the space shuttle is burning the propellant in the Solid Rocket Boosters, or SRB, which supplies most of the thrust. This time interval is also where the space shuttle is in the denser (or thicker) part of the atmosphere. During this time, the increasing dynamic pressure (or Q -Bar or simply Q) requires that the engines throttle down to about 70% to prevent damage to the space shuttle. Once Max Q is reached, the space shuttle throttles back to 100% and the acceleration increases again.

At 120 seconds the space shuttle has burned all the propellant from the SRB and they separate from the space shuttle. The SRB supplied the main thrust thus the acceleration is much lower now. By burning more propellant in the external tank (reducing mass), the acceleration increases; and as the mass continues to decrease the acceleration increases at a faster rate until the space shuttle reaches its maximum acceleration of 3 g (29.4 m/s^2) at 450 seconds.

On the interval $[460, 500]$ the space shuttle is at max acceleration where it stays until it is ready for orbit.



The acceleration of the space shuttle drops drastically on the interval [500,520] because the main engines are cut off. The external tank will separate from the space shuttle shortly and it will soon be in orbit.

- H. Is there one regression function that best fits the acceleration data? Why or why not? What is the acceleration function that describes the interval [120, 460]? Let a stand for acceleration and t stand for time. Express the coefficients as three significant figures. Keep this equation in Y_5 on your graphing calculator.

There is not one function that would fit this data. It appears to be a piecewise function. There would be a different function for each of the intervals discussed in question 6.

To find the equation on the interval [120, 460]:

Step 1: The interval [120,460] from L1 should be entered into L5 and the corresponding data from L4 into L6. If the data files were used to download the tables originally, then these lists are already in the calculator and you can skip to step 2. If not, students may need to be guided to manipulate the stat lists as follows: highlight L5, input L1 and press ENTER. Do the same for L6 inputting L4. Then delete unwanted values from each list.

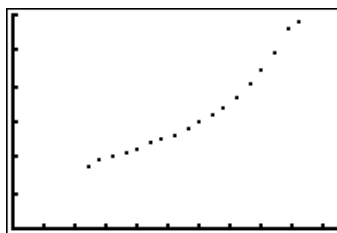
L4	4	L6	5
2.45	-----	-----	
18.62			
16.37			
19.4			
24.5			
24.01			
8.72			
L5 = L1			

L4	L5	4	6
2.45	0	-----	
18.62	20		
16.37	40		
19.4	60		
24.5	80		
24.01	100		
8.72	120		
L6 = L4			

Step 2: Change Plot 3 to plot L5 and L6 to verify the graph.

```

Plot1 Plot2 Plot3
Off Off On
Type: [ ] [ ] [ ]
Xlist:L5
Ylist:L6
Mark: [ ] [ ] [ ]
    
```



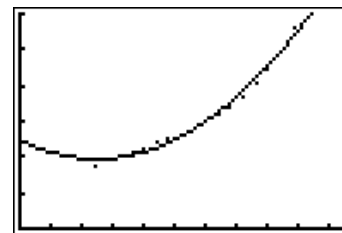
Step 3: Then calculate the quadratic regression and graph the equation to make sure it matches.

```

QuadReg (L5,L6,Y
5)
    
```

```

QuadReg
y=ax^2+bx+c
a=1.7120421E-4
b=-.0424697497
c=12.35909959
R^2=.9926090563
    
```



Rounding coefficients to six decimal places gives:

$$a(t) = 0.000171t^2 - 0.045470t + 12.359100$$

Expressing coefficients in three significant figures gives:

$$a(t) = (1.71 \times 10^{-4})t^2 - (4.25 \times 10^{-2})t + 12.4$$

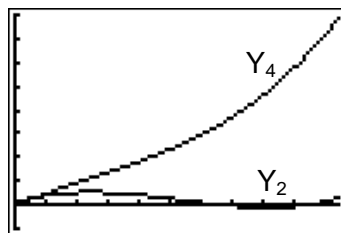


- I. Compare the graphs in Y_2 (the first derivative of the altitude function) and in Y_4 (the velocity function found by doing the regression of the data). Do this by turning all other equations off. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.

Turn off all plots and all equations but Y_2 and Y_4 , adjust window, and graph

```

WINDOW
Xmin=0
Xmax=520
Xscl=50
Ymin=-1000
Ymax=8000
Yscl=1000
Xres=1
    
```



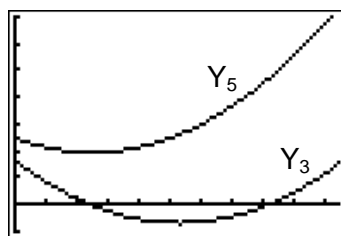
These graphs should both describe the velocity of the space shuttle, however the derivative of the altitude function explains only the vertical velocity of the space shuttle and the data given in the table is the total velocity of the space shuttle. The shuttle is not moving just vertically but at an angle in order to go into orbit. Thus there is a vertical velocity and a horizontal velocity.

- J. Now compare the graphs in Y_3 (the second derivative of the altitude function) and in Y_5 (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.

Turn off Y_2 and Y_4 and turn on Y_3 and Y_5 , adjust window, and graph.

```

WINDOW
Xmin=0
Xmax=520
Xscl=50
Ymin=-5
Ymax=35
Yscl=5
Xres=1
    
```



These graphs both describe the acceleration of the space shuttle, but again the second derivative of the altitude function describes the vertical acceleration and the data given in the table describes the total acceleration.



Scoring Guide

Suggested 21 points total to be given.

Question	Distribution of points
A <i>2 points</i>	1 point for the scatter plot 1 point for the quartic regression equation
B <i>2 points</i>	1 point for the equation 1 point for the explanation
C <i>2 points</i>	1 point for the local extrema 1 point for the explanation of what the extrema represent
D <i>2 points</i>	1 point for the inflection points 1 point for the description of concavity
E <i>1 point</i>	1 point for the conclusion of SRB separation
F <i>3 points</i>	1 point for the scatter plot 1 point for the explanation 1 point for the velocity function
G <i>6 points</i>	1 point for the scatter plot 1 point for locating maxQ 1 point for locating SRB separation 1 point for explanation of acceleration increase on [120,460] 1 point for explanation of acceleration on [480,500] 1 point for identifying main engine cut off on [500,520]
H <i>1 points</i>	1 point for the acceleration function
I <i>1 points</i>	1 point for explaining the differences in the velocity functions
J <i>1 points</i>	1 point for explaining the differences in the acceleration functions



Appendix A

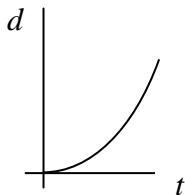
Discussion Guide for STS 121 Launch Video “The Rocket’s Red Glare”

Before playing the launch video, encourage students to pay close attention to the communication that goes on between the space shuttle crew and mission control and to strive to gain a sense of what the space shuttle is physically experiencing during the climb to orbit. After the students have watched the video discuss what they observed. The following questions and answers are just a few examples of questions that could be used in a class discussion of the video.

1. Name some of the forces that are involved in the ascent process.

Throttle down occurs as the space shuttle approaches and passes through the sound barrier where the pressure of the atmosphere produces enormous force on the space shuttle system and the crew inside the vehicle. “Accelerating more slowly” reduces these forces. Throttle up occurs after this point of maximum dynamic pressure (Max Q) has passed and allows the space shuttle to accelerate continuing its ascent.

2. After the space shuttle has cleared the tower the video image is from the Solid Rocket Booster (SRB). You can literally see Florida “fall down” below the rising space shuttle. Sketch a d - t graph of this portion of the ascent or explain what it would look like.



The graph is concave up, does not have a constant slope and its origin is at the launch pad.

3. As the Solid Rocket Boosters (SRBs) burn out and the space shuttle approaches SRB separation, what is happening to its acceleration?

As the SRBs burn out they are no longer able to contribute significantly to accelerating the space shuttle to orbit. Therefore, the space shuttle, although still accelerating due to its own engines, slows its rate of acceleration rapidly. After SRB separation there is a large jolt as the space shuttle is able to accelerate more quickly, now that it has lost a huge amount of mass.

4. Consider the statement “from zero to 17,500 mph in 8.5 min.” Ponder the enormity of these values in comparison to the fastest car you’ve ever heard of. What are the factors that contribute to the space shuttle achieving such a magnificently huge orbital speed?

Some factors include decreasing mass (propellant use and jettison of SRBs), constant thrust, and decreasing atmospheric drag.

5. Pay special attention to the camera angle provided of the Earth as it slips past in the background. How would you describe the space shuttle’s progress (constant speed, accelerating, at rest...)?

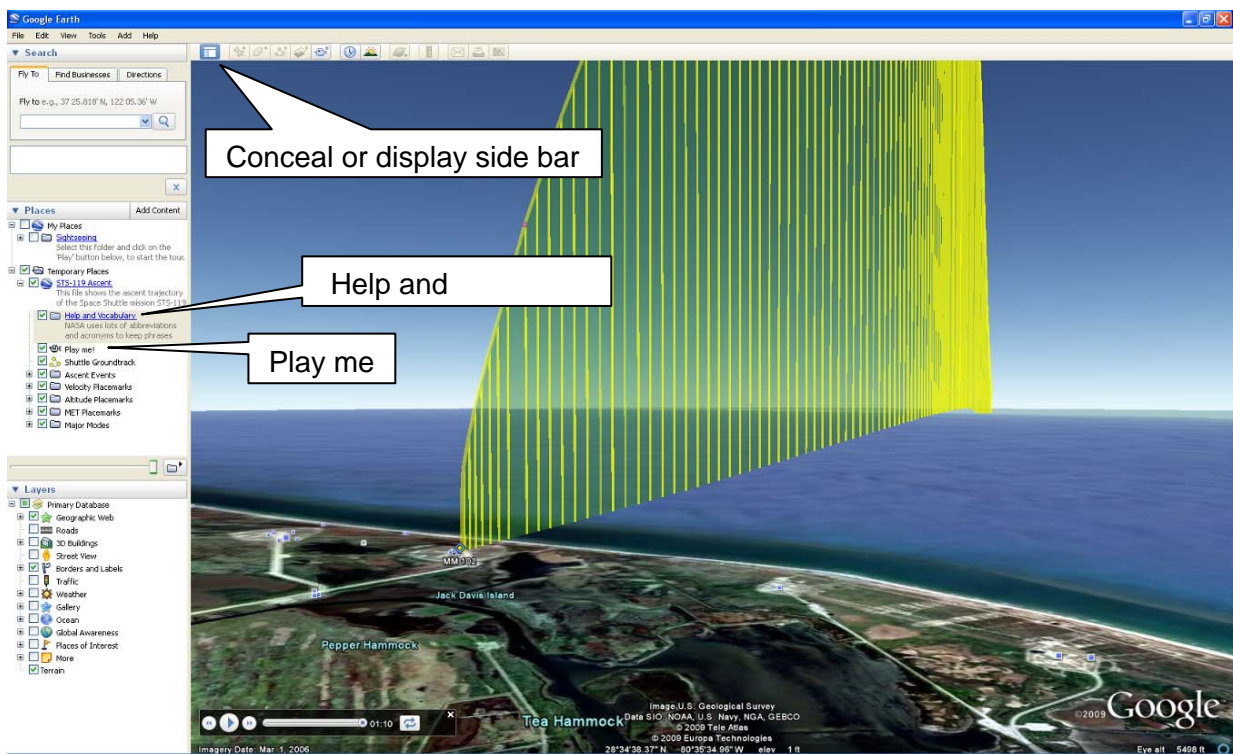
Because the space shuttle’s altitude is so great now it is nearly impossible to tell from the video that it is still accelerating. Recall, however, that the main engines are still running, propellant mass is decreasing quickly, and there is no atmospheric drag. One can conclude that the space shuttle is accelerating.



Appendix B

Instructions for Google Earth Space Shuttle Ascent Trajectory with Tour

1. Prior to presenting this file to your students, you should become familiar with its features and the features of Google Earth.
2. To access the file, you must first have Google Earth installed on the computer. To download a free version of Goggle Earth, follow the link below and choose "Download Google Earth 5". <http://earth.google.com>. The Google Earth user guide can be found under the help menu and gives a good overview of the software.
3. Download the provided file AscentTrajectory_Tour.kmz.
4. Open the file (AscentTrajectory_Tour.kmz) by selecting File > Open and select the file.



5. To learn more about this file, click on **Help and Vocabulary**, located on the left frame under **Temporary Places**.
6. An animated tour of the ascent trajectory is provided with this file and is 1:10 minutes long. To start the animation, double click on **Play Me!**
7. While watching the tour you can conceal the side bar by clicking the **Conceal** button located at the top left of the tool bar.
8. The tour begins with an overview of the entire space shuttle ascent from several different angles. This overview is 20 seconds long.
9. After the overview, the sequence will begin again. There are three 5 second pauses for inserting explanations of the ascent events: SRB Separation, MECO, and ET Separation. Each event is marked with a black and red circle.



10. There are additional features that you may want to explore with students at the conclusion of the tour. You may also want to use the following subtopics to unclutter the animation, (for example, uncheck the altitude and velocity folders).

To expand a folder in the left frame, click on the “+” next to the folder. To collapse the folder, click on the “-” next to it. For each subtopic there is a pop-up information box that can be opened by clicking on the subtopic. To close the pop-up information box, click on the “x” in the upper right corner of the box (or uncheck the small checked box to the left of the subtopic by clicking on it).

- Ascent Events Folder (expanded)
 - Click on an event and a pop-up information box will appear on the graph.
 - Notice the bullets for the events are the same as the markers (red and black circles) that appear on the graph for these events.
- Velocity Placemarks Folder (expanded)
 - Click on a velocity and a pop-up information box will appear on the graph.
 - Notice the bullets for the velocities are the same as the markers (green circles) that appear on the graph for these velocities.
- Altitude Placemarks Folder (expanded)
 - Click on an altitude and a pop-up will information box will appear on the graph.
 - Notice the bullets for the altitudes are the same as the markers (pink circles) that appear on the graph for these altitudes.
- MET (Mission Elapsed Time) Placemarks Folder (expanded)
 - Click on an MET and a pop-up information box will appear on the graph.
 - Notice the bullets for the METs are the same as the markers (orange circles) that appear on the graph for these METs.
- Major Modes (MM) Folder (expanded)
 - Click on a major mode and a pop-up information box will appear on the graph.
 - Notice the bullets for the major modes are the same as the markers (blue diamonds) that appear on the graph for these major modes.



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Thanks to the subject matter experts for their contributions in developing this problem:

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