# Which Note am I playing?

Teachers Teaching with Technology™
Professional Development from Texas Instruments

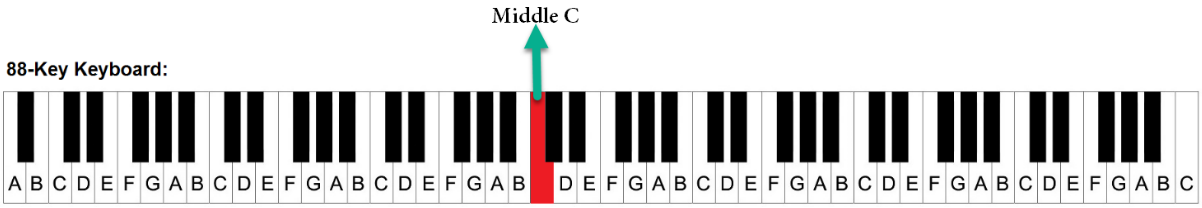
## Teacher Notes and Answers

7 8 **9** **10** 11 12

TI-Nspire CXII/CXII CAS | Investigation | Teacher | 50 min

### Which Musical Note am I playing?

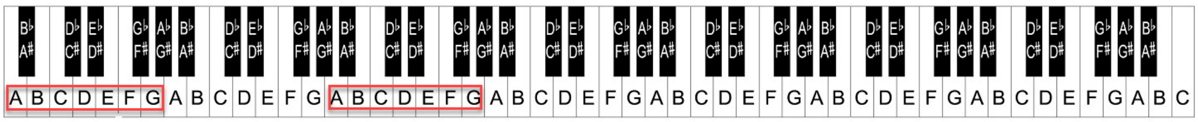
In a full Piano, there are 88 Keys. Each key plays a different note. There is a mathematical pattern associated with the note frequency as we move from left to right. The same happens with any musical instrument.




### Investigating and analysing the pattern and the note frequency.

Notes are defined by the frequency. On a Piano the key on the extreme left (1st key) plays the lowest note and the last key (extreme right plays the highest note)

88-Key Piano Keyboard Layout




Note: The first key on an 88 key piano is the A note and the last keynote is C.

**Question 1:** On an 88 Key Piano how many A note keys (amongst white keys) do you notice?

Answer: **8 Keys**

**Question 2:** On an 88 Key Piano how many C note keys (amongst white keys) do you notice?

Answer: **8 Keys**

**Question 3:** On an 88 Key Piano how many D or E or F or G note keys (amongst white keys) do you notice?

Answer: **7 Keys**

**8A+8B+8C+7D+7E+7F+7G=52 White Keys**

**36 Black Keys (Flat and Sharp Notes)**

**Total 52+36=88 Keys**



Author : S Meston

## Total Number of white and black Keys

**8A+8B+8C+7D+7E+7F+7G=52 White Keys**

**36 Black Keys (Flat and Sharp Notes)** **Total 52+36=88 Keys**

## Let's Identify the pattern

The First A note (key 1) has a Note frequency of 27.5 Hz. (Hz=Hertz is the number of cycles /second)

The key numbering now includes the White and the Black keys

**Question 4:** Complete the Table underneath with

| Key No. | Key (Note) Reference | Note Frequency in Hertz | Write a possible Recursive Pattern to determine the frequency |
|---|---|---|---|
| Key 1 | 1A | 27.5 | 27.5 or . |
| ey 13 | 2A | 55 | $2\times27.5$ or $2^1\times27.5$ or $2\times1A$ |
| Key 25 | 3A | 110 | $4\times27.5$ or $2^2\times27.5$ or $2\times2A$ |
| Key 37 | 4A | 220 | $8\times27.5$ or $2^3\times27.5$ or $2\times3A$ |
| Key 49 | 5A | 440 | $16\times27.5$ or $2^4\times27.5$ or $2\times4A$ |
| Key 61 | 6A | 880 | $32\times27.5$ or $2^5\times27.5$ or $2\times5A$ |
| Key 73 | 7A | 1760 | $64\times27.5$ or $2^6\times27.5$ or $2\times6A$ |
| Key 85 | 8A | 3520 | $128\times27.5$ or $2^7\times27.5$ or $2\times7A$ |

Note: The human audible range is 20Hz-20,000 Hz. The audible range reduces with age. Our audible hearing range typically reduces with age, so it is quite likely that elderly people may not hear frequencies over 12kHz.

**By end of this task, we should be able to work out the frequency for the 88th Key**

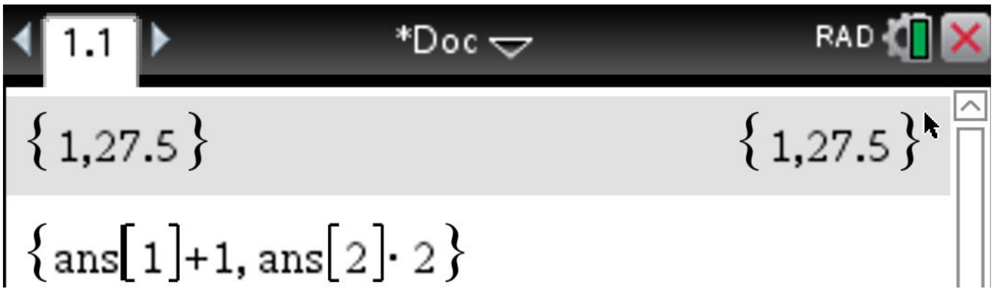
**A similar Table can be created for (B or C or D or E or F Notes).**

**Try this on your TI-Nspire**

- Enter Line 1 in Curly Brackets (Braces) and Enter
- Enter Line 2 in Curly Brackets. It uses the answer from previous line (line 1 in this case)



Author: S. Meston




**Keep hitting the enter key). Do it 7 times.**

- **The First value in the output is "A Note Reference Number" A1, A2 ………….**
- **Second value is the Corresponding frequency for the A Notes 27.5, 55 ……….**

| | |
|---|---|
| $\{1,27.5\}$ | $\{1,27.5\}$ |
| $\{\{1,27.5\}[1]+1,\{1,27.5\}[2]\cdot 2\}$ | $\{2,55.\}$ |
| $\{\{2,55.\}[1]+1,\{2,55.\}[2]\cdot 2\}$ | $\{3,110.\}$ |
| $\{\{3,110.\}[1]+1,\{3,110.\}[2]\cdot 2\}$ | $\{4,220.\}$ |
| $\{\{4,220.\}[1]+1,\{4,220.\}[2]\cdot 2\}$ | $\{5,440.\}$ |
| $\{\{5,440.\}[1]+1,\{5,440.\}[2]\cdot 2\}$ | $\{6,880.\}$ |
| $\{\{6,880.\}[1]+1,\{6,880.\}[2]\cdot 2\}$ | $\{7,1760.\}$ |
| $\{\{7,1760.\}[1]+1,\{7,1760.\}[2]\cdot 2\}$ | $\{8,3520.\}$ |

**Keep hitting the enter key). Do it 7 times.**

- **The First value in the output is "A Note Reference Number " A1, A2 ………….**
- **Second value is the Corresponding frequency for the A Notes 27.5, 55 ……….**

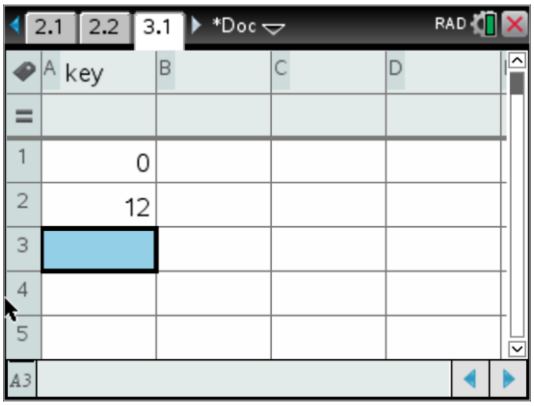


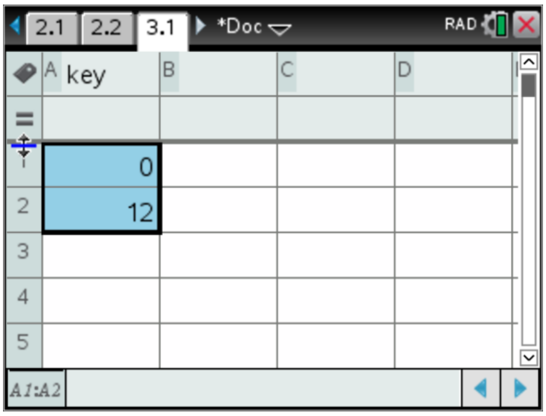
Author: S. Meston

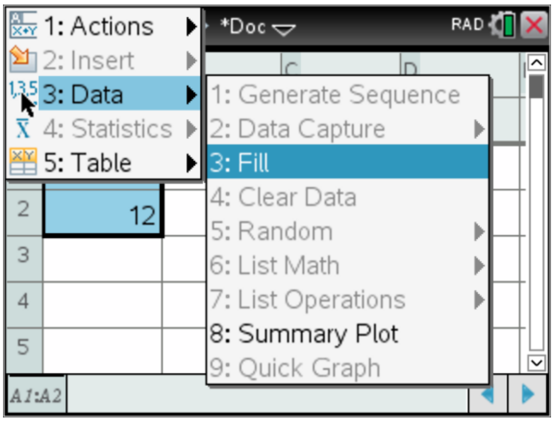
## Extension Task:

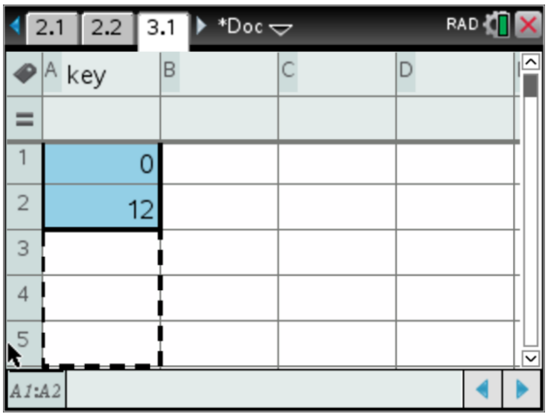
Try obtaining the same pattern on TI-Nspire

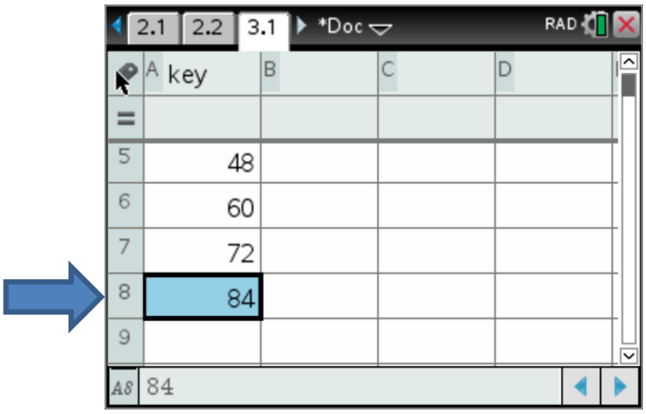
a. using the List and Spread-sheet Application



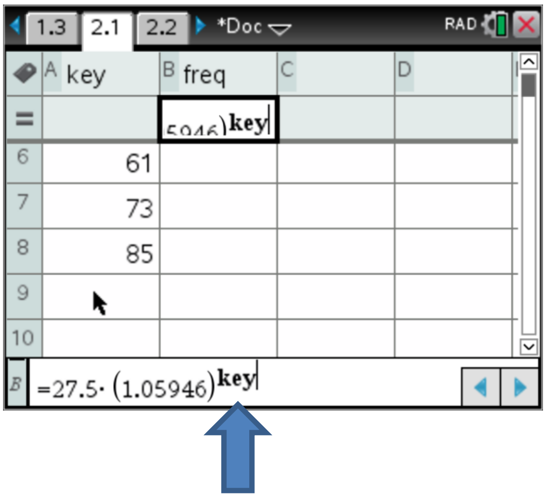



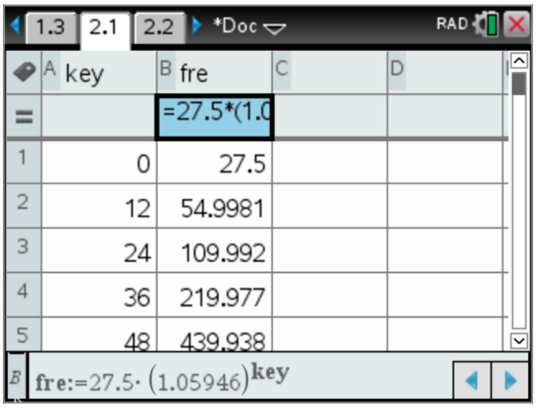

Stop at cell A-8 (Column A and Row 8)

For Frequencies in B column call it ‘freq’ and in the formula box enter

$$= 27.5 \times (1.05946)^{key}$$

Key is a reference to values from column 1, We have used an approximated value so values will be very close to 55, 110, 220, 449 etc




b. Generate a sequence (using the sequence command)

$$\text{seq}(12\cdot x+1,x,0,7) \blacktriangleright \{1,13,25,37,49,61,73,85\}$$

$$\text{seq}\left(27.5\cdot 2^{x},x,0,7\right)$$

$$\blacktriangleright \left\{\frac{55}{2},55,110,220,440,880,1760,3520\right\}$$

c. You may try to obtain the pattern in TI-Nspire using

i. TI-Basic

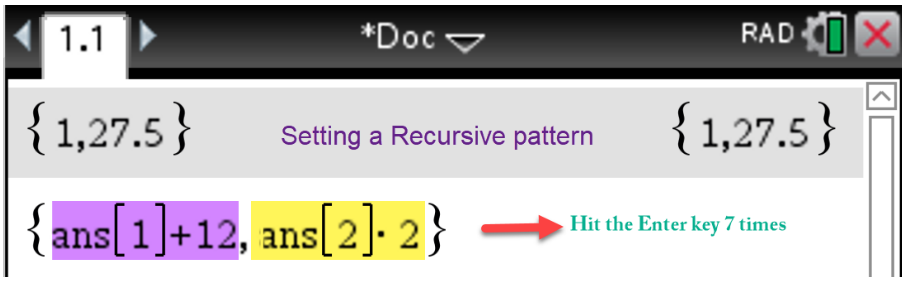
ii. Python.



Author: S. Meston

## Part 2: What is Exponential Growth and what is the Exponential Pattern for Music Notes.

### Introduction to Exponential Equations and Exponential Regression




This is the result

| | |
|---|---|
| $\{1,27.5\}$ | $\{1,27.5\}$ |
| $\{\{1,27.5\}[1]+12,\{1,27.5\}[2]\cdot 2\}$ | $\{13,55.\}$ |
| $\{\{13,55.\}[1]+12,\{13,55.\}[2]\cdot 2\}$ | $\{25,110.\}$ |
| $\{\{25,110.\}[1]+12,\{25,110.\}[2]\cdot 2\}$ | $\{37,220.\}$ |
| $\{\{37,220.\}[1]+12,\{37,220.\}[2]\cdot 2\}$ | $\{49,440.\}$ |
| $\{\{49,440.\}[1]+12,\{49,440.\}[2]\cdot 2\}$ | $\{61,880.\}$ |
| $\{\{61,880.\}[1]+12,\{61,880.\}[2]\cdot 2\}$ | $\{73,1760.\}$ |
| $\{\{73,1760.\}[1]+12,\{73,1760.\}[2]\cdot 2\}$ | $\{85,3520.\}$ |

**Question 1:** What possibly is represented by the first of the two values in the output in each line for the 8 rows?

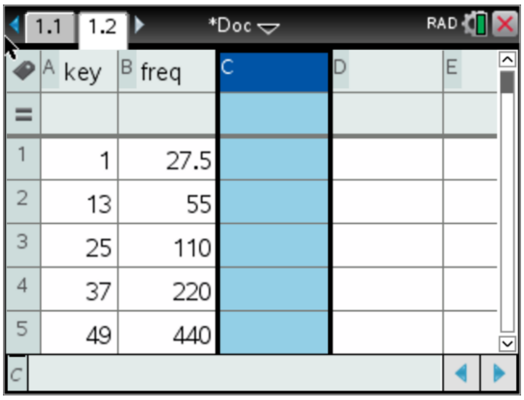
Ans: First Value represents the key number for all the A notes

### Using the List & Spread-Sheet and Data & Statistics Applications on TI-Nspire

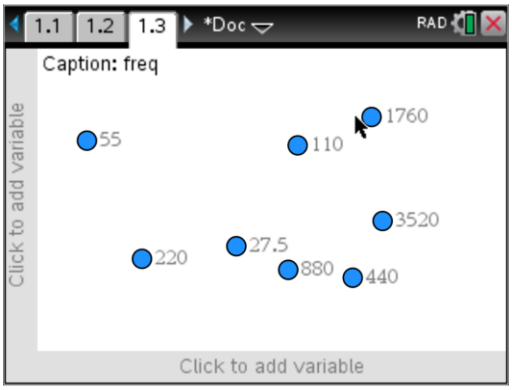
**Step 1:** Enter the Values as shown below in a **List & Spreadsheet Application**

Col A: Key {0,12,24,36,48,60,72,84}

Col B: freq {27.5,55,110,220,440,880,1760,3520}

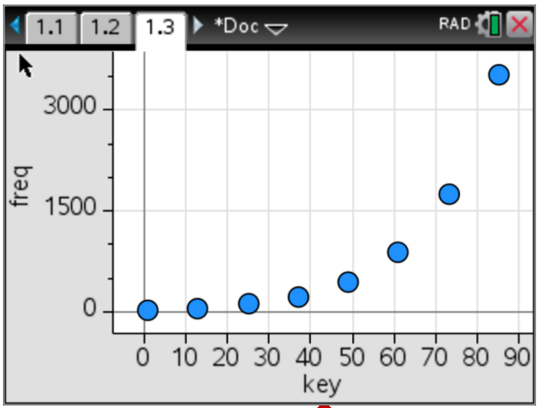


**Step 2:** Open the Data&Statistics Application

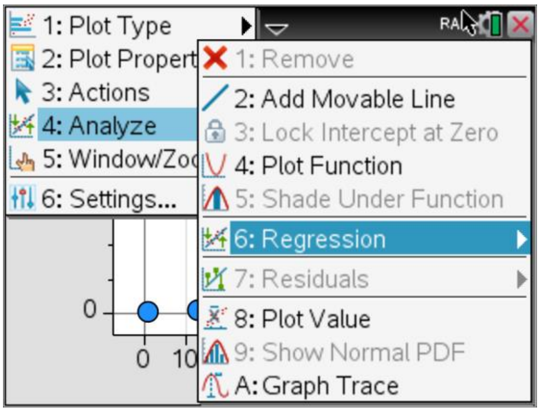


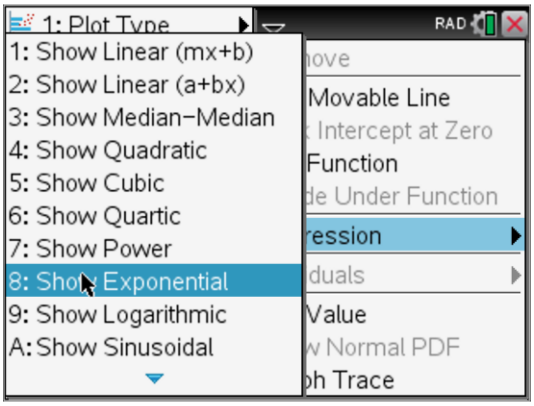

**Next Click on add variable on x-axis and select key for x-axis**

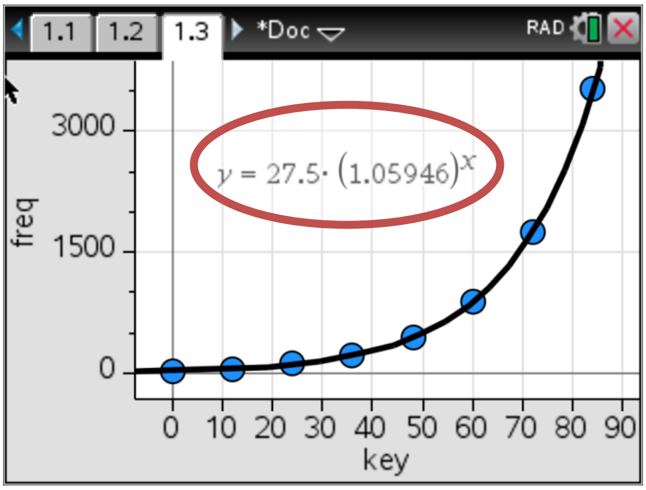
**Repeat the same to add freq as variable on y-axis**




**Step 3:** Obtaining a regression equation (Menu+Analyse) and follow the steps as under






Author: S. Meston

**Question 2:**

i. What is the value **27.5** in the regression equation?

Ans: It is the frequency of the First key A1=27.5

ii. What will; $x$ *input value represent* $in\ y = 27.5\ \times 1.05946^x$ ?

**Hint**: we started with zero and not 1 for keys,

X= 0 represents Key 1, x=1 represents Key number 2, x=87 represents key number 88

So, the A Notes are on keys {1,13,25…………85} which correspond to x= {0,12,24……….84}

Ans: x+ 1 will be the key number

i. What information will; $y$ *output value represent* $in\ y = 27.5\ \times 1.05946^x$

when x Є {0,1,2,3…….87}

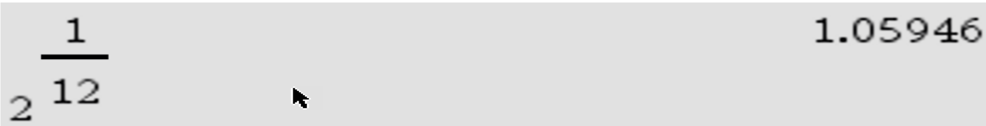
Ans: y represents the Note frequency for a key given by x+1

ii. Do you want to guess what the value 1.0594 may be??? You'll find the answer at the end of this worksheet.

Ans: Since there are 12 keys between the A notes. $2^{\frac{1}{12}} \approx 1.05946$ . 1.0594 is the multiplying factor to obtain the note frequency for the next key. Since the Note frequency doubles (x2) from one A note to the next A Note.

Example: The Note frequency for the 23rd key will be

$$F_{23} = 1.0594 \times F_{23}$$




**Question 3:** Using the Equation $y = 27.5\ \times 1.05946^x$

For this question express your answers to 2 decimal places.

a. Find the Note frequency for the 14th Key (Hint: This key is not an A Note)

58.27 Hz

b. Find the Note Frequency for the 88th Key (Last key on the Piano). Note this is a C Note

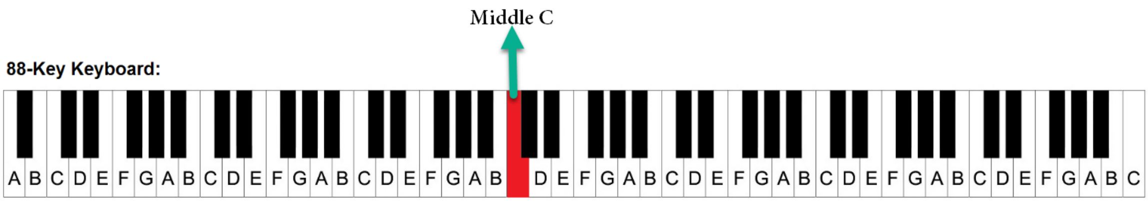
4184.95 Hz

c. For Musicians Middle C is an important note. On an 88 key Piano it is the 40th Key (including white and black keys). Determine the Note frequency for the Middle C Note

261.60 Hz



Author: S. Meston




$y=27.5\cdot(1.05946)^x|x=13$ 14th Key $y=58.2683$

$y=27.5\cdot(1.05946)^x|x=87$ 88th key $y=4184.95$

$y=27.5\cdot(1.05946)^x|x=39$ 40th key $y=261.596$

**Question 4:** Using the Equation $y = 27.5 \times 1.05946^x$

a. Complete this table for the first 12 keys (This includes the white and the Black Keys)

The lowest note on the 88 Piano key is 27.5 Hz and corresponds to $A_1$ (key number 1)

The table on the next page is for the first 12 keys of the Piano. You need to complete it for Keys 6-12

The table underneath is for the first 12 keys of the Piano

| Note | A | A# | B | C | C# | D | D# | E | F | F# | G | G# |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Key (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Freq in Hz | 27.500 | 29.135 | 30.867 | 32.703 | 34.647 | | | | | | | |
| Key Colour | | | | | | | | | | | | |

The values are rounded to 2 decimal places

$$y=27.5\cdot(1.05946)^x|x=\{5,6,7,8,9,10,11\}$$
$$=\{36.7076,38.8902,41.2026,43.6525,46.2481,48.998,51.9114\}$$

$$y=\text{round}(\{36.7076,38.8902,41.2026,43.6525,46.2481,48.998,51.9114$$
$$y=\{36.71,38.89,41.2,43.65,46.25,49.,51.91$$

D=36.71

D#=38.89

E=41.20

F=43.65

F#=46.25

G=49.00

G#=51.91



Author: S. Meston

b. Using the table values state the ratio for the following to two decimal places

$\frac{Key2}{Key1} = \boxed{1.06}$ $\frac{Key4}{Key3} = \boxed{1.06}$ $\frac{Key12}{Key11} = \boxed{1.06}$

c. Hence using the ratio value, develop a recursive pattern for two consecutive keys to obtain the frequency of $Key_{n+1}$ in terms of $Key_n$ ($Key_n$ is the preceding key to $Key_{n+1}$)

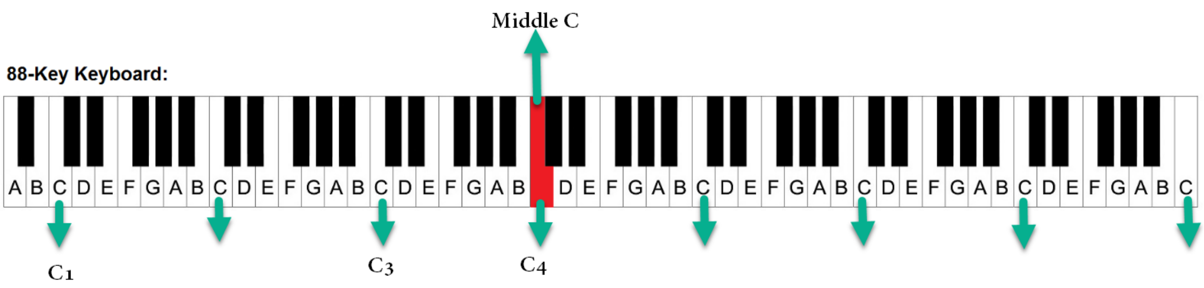
$\mathrm{F}(Key_{n+1}) = 1.06 \times \mathrm{F}(Key_n)$ where F is the frequency



Author: S. Meston

## Part 3: This section is Meant for students in the Year 10 Advanced Mathematics course

## Understanding Octaves and Exponential Equations




**Question 1:**

a. On an 88 key Piano, how many keys can play the C note?

   8 Keys

b. What would be a quick way to Identify the C note key on a Piano in relation to the black keys?

   The C key is always the white Key before the pair of Black keys (the two black keys)

c. Ignoring the first black key, what pattern do you observe with the black keys?
   Two black keys with a white key in between, followed by 2 white keys, then 3 black keys again with a white key in between each of them and then 2wo white keys again before the pattern repeats.

d. What will be a quick way to identify the B note on a Piano keyboard?

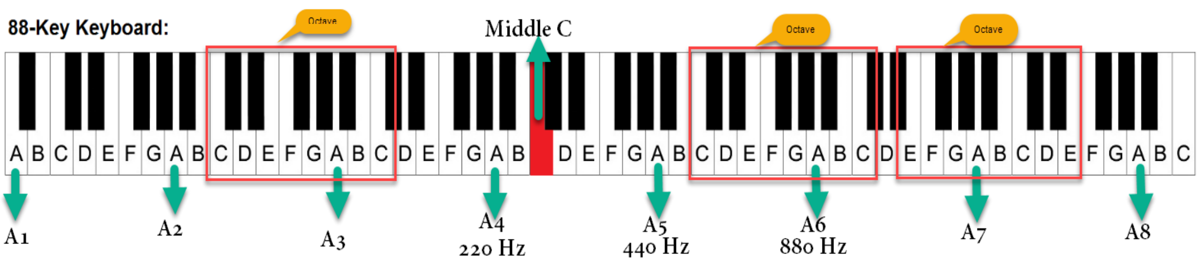
   B key is always the white key after the group of 3 black keys (white key after the black triad)

**Octave:** An octave includes 12 keys between two musical notes that have the same letter Note.

It is called an 'octave' because there are eight notes in a scale ('octo' is Latin for 'eight')

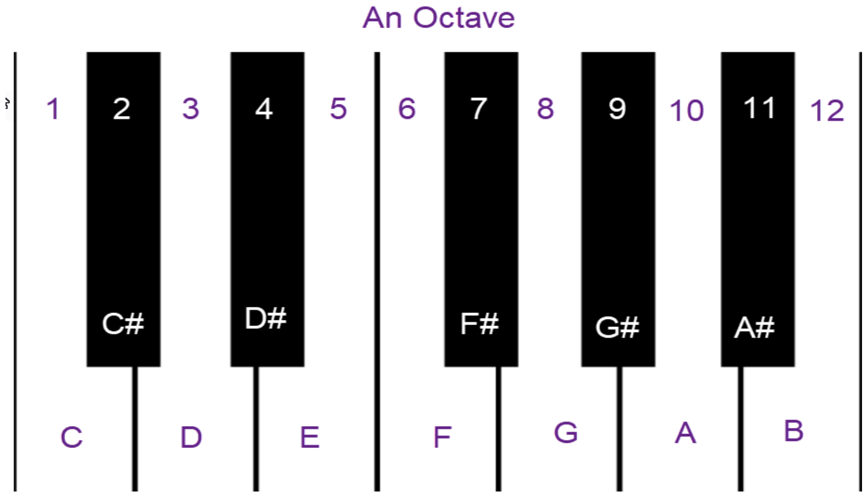
The white keys unnatural notes are assigned letters A to G. The Black Keys are assigned the letter symbol followed by a sharp (symbol) or a flat (symbol) so if we are moving from left, the black key to the right of C would be C sharp and the same black key which is also to the left of D can be classified as D flat so C sharp and D flat will have the same frequency and are the same key

Likewise, the next black key (in the group of Black keys) will be D sharp and E Flat






Author: S. Meston




**Question 2**

A Mathematical rule to determine the frequency of the A notes is $A(n) = 27.5 \times 2^{n-1}; n \in Z$ and $1 \le n \le 8$

**a.** Explain how this rule may have been obtained

27.5 is the frequency of the 1st Key
2 because the frquency of notes doubles each time
$2^{n-1}; 1 \le n \le 8$ since key 1 is n=1, and the first power should be zero, $\because$ $2^0 = 1$

**b.** Using the same Mathematical logic state a rule to obtain the frequency for all the Eight C Notes on the Piano in the form $C(n) = F \times 2^{n-1}; n \in Z$ and $1 \le n \le 8$. You may need to obtain data values from the table you completed in the previous section

$C(n) = 32.703 \times 2^{n-1}; n \in Z$ and $1 \le n \le 8$

**Considering the fact, that there are 12 keys in an octave, we will modify the rule**

$A(n) = 27.5 \times 2^{n-1}; n \in Z$ and $1 \le n \le 8$

to obtain the frequency for the first 12 keys

**Question 3:**

a. Write your rule in the form $F(n) = 27.5 \times 2^{\frac{n-1}{b}}; n \in Z$ and $1 \le n \le 12$

by assigning a numeric value to b. Explain how you obtained the value of b.

$F(n) = 27.5 \times 2^{\frac{n-1}{12}}; n \in Z$ and $1 \le n \le 12$
Reason: Since there are 12 keys in a Octave.

so the note frequency increases by a factors of $2^{\frac{1}{12}}$ for each subsequent key

b. Modify your rule to obtain the Note frequency for all the 88 Keys on a Piano

$$F(n) = 27.5 \times 2^{\frac{n-1}{12}}; n \in Z \text{ and } 1 \leq n \leq 12$$

Concluding Remarks

Not all Keyboards have 88 Keys; hence the first key will not always be 27.5 Hz, therefore the Mathematical rule obtained by you is modified to make 440 as the principal frequency.

$$f(n) = 440\left(2^{\frac{n-49}{12}}\right) \quad ; 1 \leq n \leq 88$$

And the answer to the guessing question is

| | |
|---|---|
| $2^{\frac{1}{12}}$ | 1.05946 |

The frequency table or chart works for all musical instruments, except for the facts that some musical instruments have fewer octaves