Math Objectives

- Students will identify lines with zero and undefined slope.
- Students will describe, in terms of coordinates, what is special about points on horizontal and vertical lines.
- Students will explain why the slope of a horizontal line is zero and why the slope of a vertical line is undefined.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practices).

Vocabulary

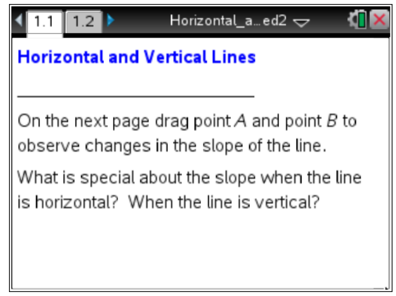
- horizontal lines
- slope
- vertical lines

About the Lesson

- This lesson involves examining the vertical and horizontal changes when moving from one point to another on a line.
- As a result, students will:
  - Understand that when the vertical change between two distinct points is zero, the slope is zero, and when the horizontal change between two distinct points is zero, the slope is undefined.
  - Move points and observe the line move in relation to the vertical and horizontal changes.

TI-Nspire™ Navigator™ System

- Use Quick Poll to share students' responses and assess students' understanding.
- Use Live Presenter to examine vertical change, horizontal change, and slope.
- Use Screen Capture to discuss characteristics of lines.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.

Lesson Materials:
Student Activity
Horizontal_and_Vertical_Lines_Student.pdf
Horizontal_and_Vertical_Lines_ Student.doc

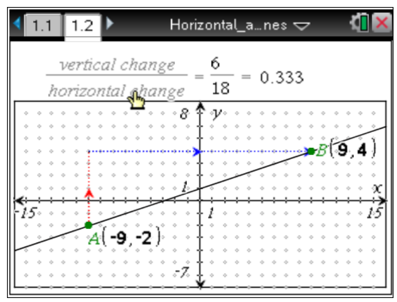
TI-Nspire document
Horizontal_and_Vertical_Lines.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty moving a point, move the cursor to hover over the point until a pair of horizontal and vertical arrows appears. The right, left, up and down directional arrows on the touch pad will move the point on grid points. Students might also drag the point. Check to make sure that they have moved the cursor (arrow) until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press [ctrl] [click] to grab the point and close the hand (✊). When finished moving the point, press [esc] to release the point. Once a function has been graphed, the entry line can be shown by pressing [ctrl] [G]. The entry line can also be expanded or collapsed by clicking the chevron.

Move to page 1.2.



1. a. Following the arrows from point *A* to point *B*, what is the value of the vertical change?

   Answer: The number of units counted up/down on the triangle (height of the slope triangle). On page 1.2 shown to the right as it opens, the vertical change is 8.

   b. Horizontal change?

   Answer: The number of units counted left/right on the triangle (length of the slope triangle). On page 1.2 shown above as it opens, the horizontal change is 18.

   Teacher Tip: The main point here is to focus on the amount of vertical and horizontal changes. Students can determine this by simply counting the number of units. Some students might say something about subtracting the coordinates. If so, this would be a good opportunity to discuss the slope formula. If students want to only talk about looking at the fraction at the top of the screen, emphasize counting to determine each change.

   c. What number represents the slope of the line through points *A* and *B* on page 1.2?

   Answer: The slope of the line is 4/9.

Teacher Tip: This might be an opportunity to discuss the fact that the slope is a real number and is often expressed as a number over 1, i.e. $\frac{0.6}{1}$.
Note also that in some cases as points *A* and *B* are moved, the slopes calculated from the ratio of the vertical to horizontal change is approximate rather than exact.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 1 at the end of this lesson.

2. Move one or both points until the value for the vertical change is zero and the value for the horizontal change is not zero.
   a. Describe the line passing through points *A* and *B*.

   Answer: The line is horizontal.

   Teacher Tip: The point of this question is to identify the main characteristic of a line with a zero slope. Because the vertical change is zero, the line neither rises nor falls to the left.

   Tech Tip: It is possible to move point *A* or *B* outside of the grid of the coordinate plane. If this happens, simply move the point(s) back to the coordinate plane.

   b. What is the slope of the line passing through points *A* and *B*?

   Answer: The slope of the line is 0.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 2 at the end of this lesson.

3. Move the points until the value for the horizontal change is zero and the value for the vertical change is not zero.
   a. Describe the line passing through points *A* and *B*.

Answer: The line is vertical.

Teacher Tip: The point of this question is to identify the main characteristic of a line with undefined slope. Because the horizontal change is zero, the line does not move from left to right. Slope only makes sense as the rate at which a line rises or falls per unit to the right. If a line does not move from left to right, slope has no meaning. There is also the mathematical implication of a horizontal change of zero. Since slope is defined as the ratio of the vertical change to the horizontal change, the ratio is undefined if the horizontal change is zero.

b. What is the slope of the line passing through points *A* and *B*?

Answer: The slope of the line is undefined.

Teacher Tip: You might ask students what happens when the two points coincide. The slope becomes undetermined because the line has disappeared. Be sure students recognize that it takes two points to have a line and when the points coincide there is only one point.

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)***
See Note 3 at the end of this lesson.

4. a. Why is the slope undefined when the horizontal change is zero?

Answer: The *x*-coordinates are the same, and this causes the denominator in the ratio to equal 0. Division by 0 is not possible.

b. How is this different from a zero slope?

Answer: When the slope is 0, the *y*-coordinates are the same, but the *x*-coordinates are not. The numerator is 0, and the denominator is not 0, so the quotient is 0.

Teacher Tip: Why you can't divide by zero: Every true division has a corresponding true multiplication associated with it. For example, $18 \div 3 = 6$, because $6 \times 3 = 18$. Similarly, if $5 \div 0 = a$, then $a \times 0 = 5$. This is not possible, because there is no number that multiplies with 0 to get 5. From another perspective, division is repeated subtraction: 6 divided by 2 is 3 because 6 – 2 – 2 – 2 is zero. You can subtract 2 three times from 6. 6 divided by 0 would be looking for the number of times you can subtract 0 from 6, which is not countable, so it is undefined.

5. Set point *A* to (-4, 2). Where must you move point *B* for the vertical change to be zero and the horizontal change not be zero?

Answer: *B* is to the right or left of *A* and neither above nor below *A*.

Teacher Tip: Students should observe that the *y*-coordinates are the same. This could be an opportunity to discuss the special form the equation of a horizontal line has: in this case, $y = 2$ since all the points have a common *y*-coordinate of 2.

6. Set point *A* to (-4, 2). Where must you locate point *B* for the horizontal change to be zero and the vertical change not be zero?

Answer: *B* is above or below *A* and neither to the right or left of *A*.

Teacher Tip: Students should observe that the *x*-coordinates are the same. This could be an opportunity to discuss the special form the equation of a vertical line has: in this case, $x = -4$ since all the points have a common *x*-coordinate of -4.

7. Suppose you have two points with the same *x*-coordinates. What do you know about the line through those points and about the slope ratio? Explain your reasoning.

Sample Answers: It is a vertical line. The slope ratio is undefined. I know because I tried it. And then I figured that the ratio had a zero in the denominator, so there was no horizontal change and no slope.

Teacher Tip: Students might be asked to check their answers by using the .tns file if they do not think of doing so. Try to push for more general explanations rather than justifying their answer with one or two examples.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.

8. Suppose you have the same *y*-coordinates instead of the same *x*-coordinates. Would your answer from question 7 change? Why or why not?

   Sample answer: Yes, the answer will change. It is a horizontal line. The slope ratio is 0 because the vertical change is 0 and when a fraction has 0 in the numerator and any non-zero value in the denominator, the value of the fraction is 0.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 5 at the end of this lesson.

9. a. Suppose you have a horizontal line through (5,-3). What can you say about the coordinates of the points on this line? Explain your thinking.

   Sample answer: All points on the line must have a *y*-coordinate of -3. A horizontal line has slope 0, and in order to get a 0 from the slope ratio, the *y*-coordinates in the numerator must be the same.

   b. Suppose you have a vertical line through the same point (5,-3). What can you say about the coordinates of the points on this line? Explain your thinking.

   Sample answer: All of the points on the line must have an *x*-coordinate of 5. A vertical line has undefined slope, and in order to get an undefined slope from the slope ratio, the *x*-coordinates in the denominator must be the same.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 6 at the end of this lesson.

10. What relationship exists between the coordinates of points *A* and *B* when the slope of the line passing through them is zero? Undefined?

Answer: When the *y*-coordinates are the same but the *x*-coordinates are different, then the slope is zero. When the *x*-coordinates are the same and the *y*-coordinates are different, the slope is undefined.

11. What does it mean when someone says that a line has no slope?

Answer: A vertical line does not have a slope when the *x*-coordinates are the same and the *y*-coordinates are different, because the horizontal change difference will be 0. This will give a ratio with 0 in the denominator, which is undefined.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to:

- Differentiate between zero and undefined slope.
- Identify the relationships between the coordinates of points on a line and zero and undefined slopes.
- Understand that the slope of a horizontal line is zero and the slope of a vertical line is undefined.

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Note 1

Question 1, *Quick Poll* (*Open Response*): Send three Open Response Quick Polls, asking students to submit their responses to questions 1a, 1b, and 1c. If students struggle to identify the values of the vertical change, horizontal change, and slope, use Live Presenter to identify them as a class.

Note 2

Question 2, *Quick Poll* (*Open Response*): Send an Open Response Quick Poll, asking students to send their answers to questions 2a and 2b. If students struggle describing the line or identifying the slope, take a Screen Capture and briefly discuss the orientation of the lines and value of the slope. Emphasize using the word “horizontal” to describe the lines.

Note 3

Question 3, *Quick Poll* (*Open Response*): Send an Open Response Quick Poll, asking students to send their answers to questions 3a and 3b. If students struggle describing the line or identifying the slope, take a Screen Capture and briefly discuss the orientation of the lines and value of the slope. Discuss how the orientation and slope are different from those in question 2, emphasizing the word "vertical" to describe the new lines.

Note 4

Question 7, *Screen Capture*: Discuss question 7 by asking students to move points *A* and *B* so their *x*-coordinates are the same. Take a Screen Capture, and ask students, "What do the lines have in common? What do the slope ratios have in common?" Students should identify that the lines are vertical, the slope ratios all have zero in the denominator, and the value of the slope ratios is undefined.

Note 5

Question 8, *Screen Capture*: Discuss question 8 by asking students to move points *A* and *B* so their *y*-coordinates are the same. Take a Screen Capture, and ask students, "What do the lines have in common? What do the slope ratios have in common?" Students should identify that the lines are horizontal, the slope ratios all have zero in the numerator, and the value of the slope ratios is zero.

Note 6

Question 9, *Live Presenter*: Discuss students' responses to questions 9a and 9b by selecting a Live Presenter and asking the student to move point *A* to (5, -3). Then ask the student presenter to move point *B* so the line is horizontal. Students learned from questions 7 and 8 that whenever a line is vertical or horizontal, one set of coordinates will always be equivalent. Ask the class, "When a line is horizontal, are its *x*-coordinates or *y*-coordinates equivalent? Why" Use the slope ratio to briefly examine why the *y*-coordinates of a horizontal line are equivalent. Then ask the student presenter to move point *B* so the line is vertical. Ask the class, "When a line is vertical, are its *x*-coordinates or *y*-coordinates equivalent? Why?" Use the slope ratio to briefly examine why the *x*-coordinates of a vertical line are equivalent