



### About the Lesson

In this activity, students will construct figures that prove the Pythagorean Theorem in two different ways. As a result, students will:

- Construct and measure the side lengths of several right triangles and conjecture a relationship between the areas of the squares drawn on each side.
- Prove and apply the Pythagorean Theorem.

### Vocabulary

- Pythagorean Theorem

### Teacher Preparation and Notes

- This activity is designed to be used in a high school or middle school geometry classroom.
- The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs. This can be expressed as  $c^2 = a^2 + b^2$  where  $c$  is the length of the hypotenuse.
- Depending on student skill level, you may wish to download the constructed figures to student calculators. If the files are downloaded, skip the construction steps for each problem and begin each at Step 10.

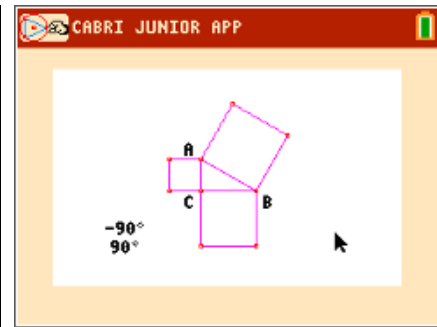
**Note:** Measurements can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:

1. Move the cursor over the value so it is highlighted.
2. Press  $\oplus$  to display additional decimal digits or  $\ominus$  to hide digits.

### Activity Materials

- Compatible TI Technologies:
  - TI-84 Plus\*
  - TI-84 Plus Silver Edition\*
  - TI-84 Plus C Silver Edition
  - TI-84 Plus CE

\* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



### Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

### Lesson Files:

- Pythagorean\_Theorem\_Student.pdf
- Pythagorean\_Theorem\_Student.doc
- PYTHAG1.8xv
- PYTHAG2.8xv



**Tech Tip:** Before beginning the activity, the files PYTHAG1.8xv and PYTHAG2.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

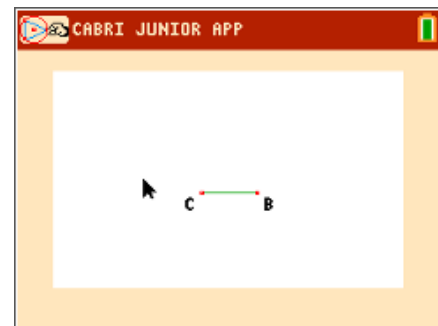
## Problem 1 – Squares on Sides Proof

The Pythagorean Theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. In this activity, you will construct a right triangle and verify the Pythagorean Theorem by constructing squares on each side and comparing the sum of the area of the two smaller squares to the area of the square of the third side.

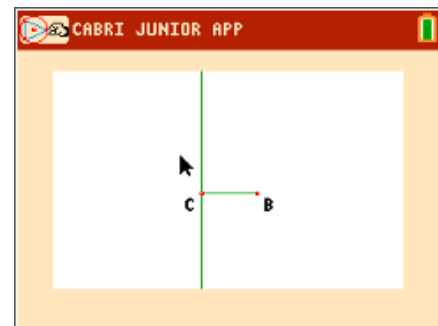
**Step 1:** Students should open a new Cabri Jr. file.

Construct a segment using the **Segment** tool.

Select the **Alph-Num** tool to label the endpoints  $B$  and  $C$  as shown.



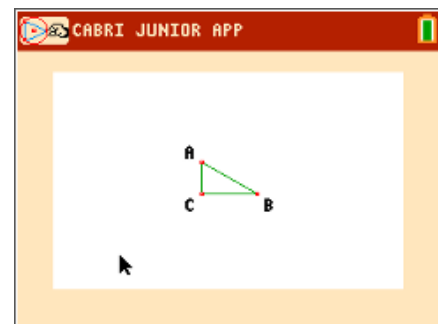
**Step 2:** Construct a line through  $C$  that is perpendicular to  $\overline{BC}$  using the **Perp.** tool.



**Step 3:** Construct a point on the perpendicular line and label it  $A$ .

Hide the perpendicular line with the **Hide/Show** tool and construct line segments  $\overline{AC}$  and  $\overline{AB}$ .

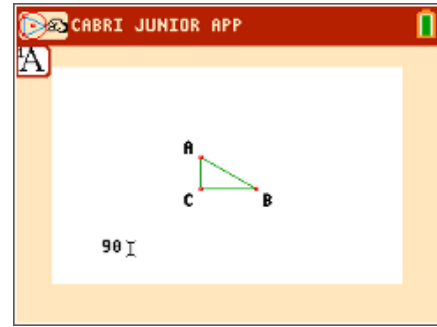
For the time being, keep the sides of the triangle fairly small so that squares can be constructed on the sides.





**Step 4:** In the lower left corner, use the **Alpha-Num** tool to place the number 90 on the screen. This will be the angle of rotation.

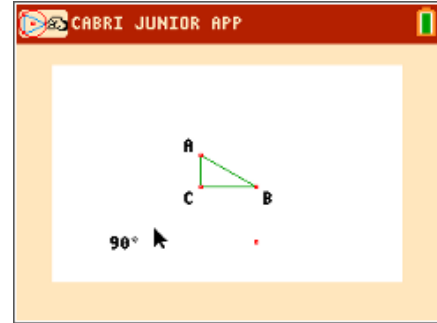
**Note:** Press  $\alpha$  to access numerical characters. A small "1" will appear in the tool icon in the upper left corner of the screen.



**Step 5:** Use the **Rotation** tool to rotate point  $C$  about point  $B$  through an angle of  $90^\circ$ .

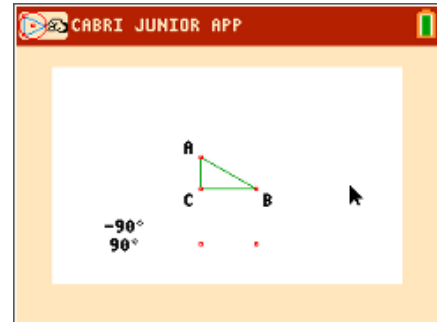
- Press  $\text{enter}$  on point  $B$  as the center of rotation.
- Press  $\text{enter}$  on the angle of rotation (the number 90).
- Press  $\text{enter}$  on point  $C$ , the object to be rotated.

Notice that the number now has a degree symbol associated with it and that the point has been rotated in the counter-clockwise direction.



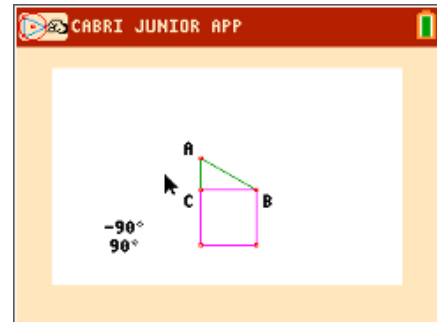
**Step 6:** What we want to do next is to rotate point  $B$  about point  $C$  through an angle of  $90^\circ$  in the clockwise direction. To do this we will need an angle of  $-90$ . Place this number on the screen.

Using the value of  $-90$ , rotate point  $B$  about point  $C$  through an angle of  $-90^\circ$ .



**Step 7:** You should now have two points below the line segment  $\overline{BC}$ . Use the **Quad.** tool to construct a quadrilateral using points  $B$ ,  $C$  and the two points constructed in Steps 5 and 6.

Can you explain why this is a square?



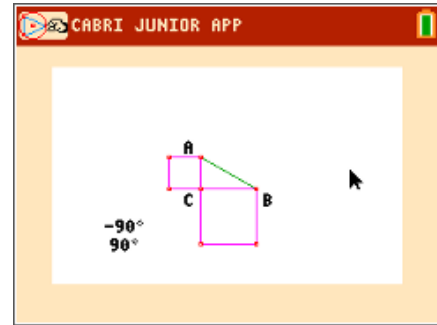
1. Why is the constructed quadrilateral a square?

**Sample Answer:** All of the sides are the same length and all of the angles are  $90^\circ$  by construction.



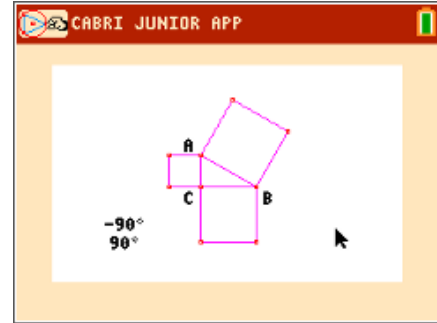
**Step 8:** In a similar fashion, rotate point  $C$  about point  $A$  through an angle of  $-90^\circ$  and rotate point  $A$  about point  $C$  through an angle of  $90^\circ$ . This will allow us to construct a second square.

Use the **Quad.** tool again to construct the square on side  $\overline{AC}$ .



**Step 9:** Finally, rotate point  $B$  about point  $A$  through an angle of  $90^\circ$  and rotate point  $A$  about point  $B$  through an angle of  $-90^\circ$ .

Then construct a third square on hypotenuse  $\overline{AB}$ .

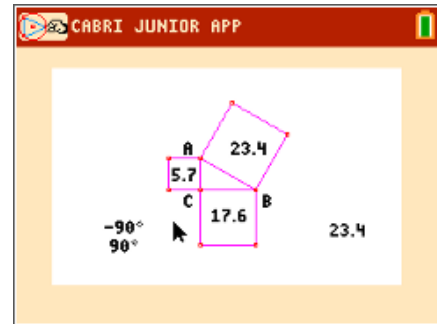


**Step 10:** Start with this step if you are using the pre-constructed file Pythag1.

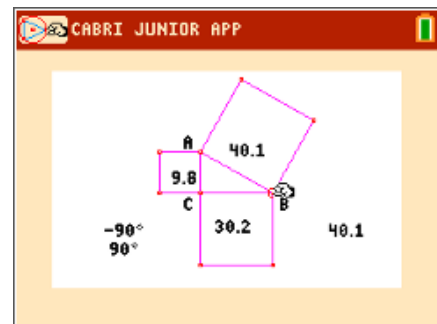
Select the **Measure > Area** tool and measure the area of the three squares.

**Step 11:** Using the **Calculate** tool, press **enter** on the measurements of the two smaller squares and then press the **+** key. Place the sum off to the side of the screen.

How does this sum compare to the square of the hypotenuse? Record your observations on the worksheet.



**Step 12:** To test your construction, drag points  $A$ ,  $B$  and/or  $C$  to a new location on the screen. Does the Pythagorean relationship still hold?





2. Record three sets of area measurements you made by dragging points  $A$ ,  $B$ , and/or  $C$ .

**Sample Answers:**

Square on $\overline{BC}$	Square on $\overline{AC}$	Sum of squares	Square on $\overline{AB}$
6.8	2.2	9.0	9.0
1.4	4.4	5.8	5.8
4.2	2.4	23.4	23.4

3. What conjecture can you make about the areas of the three squares? Does this relationship always hold when a vertex of  $\triangle ABC$  is dragged to a different location?

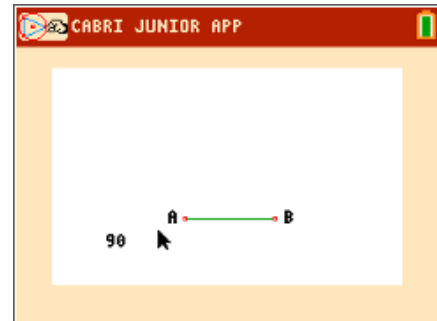
**Answer:** The sum of the areas of the square on  $\overline{BC}$  and the square on  $\overline{AC}$  are always equal to the area of the square on  $\overline{AB}$ .

### Problem 2 – Inside a Square Proof

In this problem, we are going to look at a proof of the Pythagorean Theorem. We hope to prove the statement that, if  $z$  is the length of the hypotenuse of a right triangle and  $x$  and  $y$  are the lengths of the legs of the right triangle, then  $z^2 = x^2 + y^2$ .

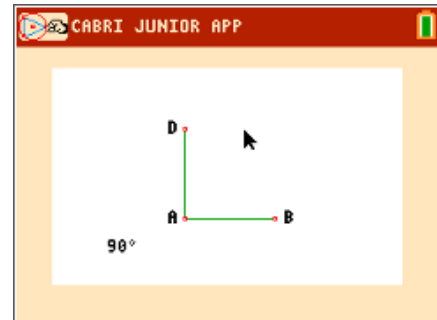
**Step 1:** Construct a line segment  $\overline{AB}$ .

Use the **Alph-Num** tool to place the value 90 on the screen.



**Step 2:** Access the **Rotation** tool and press **enter** on point  $A$  as the center of the rotation, then on 90 as the angle of rotation and finally on line segment  $\overline{AB}$  as the object to be rotated.

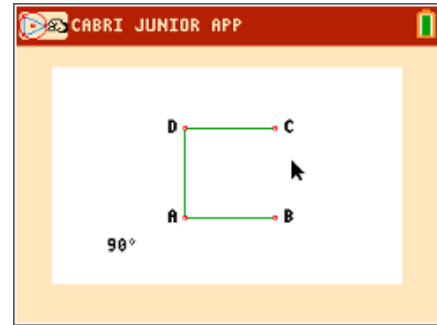
Label the new point  $D$ .





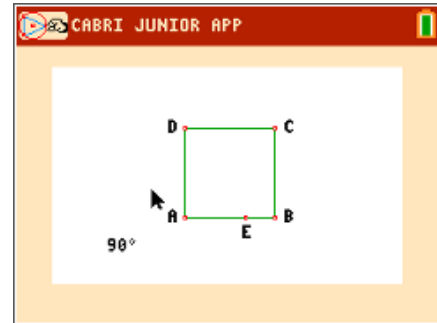
**Step 3:** Continue by rotating line segment  $\overline{AD}$  about point  $D$  through an angle of  $90^\circ$ .

Label the new point  $C$ .



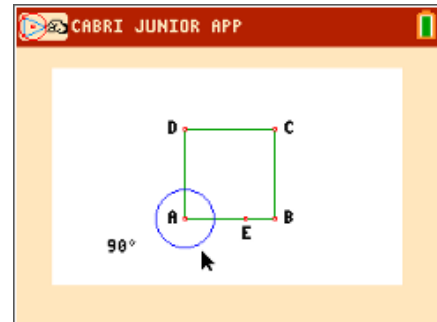
**Step 4:** Complete the square by constructing line segment  $\overline{BC}$ .

**Step 5:** Using the **Point On** tool, add point  $E$  on  $\overline{AB}$  as shown and overlay a line segment  $\overline{BE}$ .



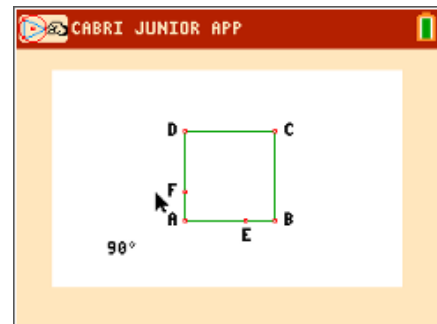
**Step 6:** Select the **Compass** tool to construct circles with radius equal to the length of  $\overline{BE}$ .

- Press **enter** on  $\overline{BE}$ . A dashed circle will appear and follow the pointer.
- Press **enter** on point  $A$ . The compass circle is anchored at center  $A$ .



Create a point of intersection of this circle with  $\overline{AD}$ . Label this point  $F$ .

Hide the compass circle.



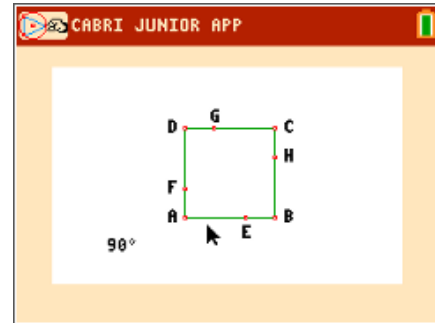
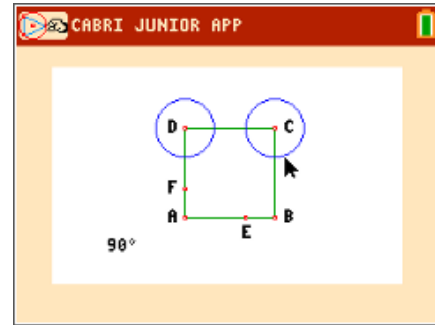


**Step 7:** Use the **Compass** tool again to construct circles with centers at  $C$  and  $D$  and radius =  $BE$ .

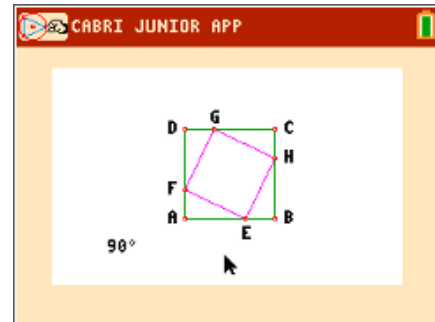
Create points of intersection of these circles with  $\overline{DC}$  and  $\overline{BC}$ . Label these points  $G$  and  $H$ .

Hide the compass circles.

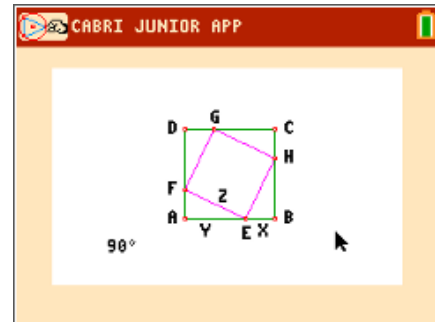
Drag point  $E$  to confirm that  $F$ ,  $G$ , and  $H$  all move as  $E$  moves.



**Step 8:** Construct the quadrilateral  $EFGH$ . Can you prove that this quadrilateral is a square?



**Step 9:** Use the **Alph-Num** tool to place the labels  $x$ ,  $y$ , and  $z$  on the figure as shown.





4. Prove that constructed quadrilateral  $EFGH$  is a square.

**Sample Answer:**

$\overline{BE}$  is labeled  $x$ ; therefore,  $AF = DG = CH = x$ .

$\overline{EA}$  is labeled  $y$ ; therefore,  $FD = GC = HB = y$ .

Because  $ABCD$  is a square, each of the angles at  $A$ ,  $B$ ,  $C$ , and  $D$  are  $90^\circ$ . So, we have four congruent triangles, namely  $\triangle EFA$ ,  $\triangle FGD$ ,  $\triangle GHC$ , and  $\triangle HEB$ .

**Step 10:** *Start with this step if you are using the pre-constructed file Pythag2.*

Let's examine the algebra in this situation.

$ABCD$  is a square with sides of length  $(x + y)$ .

The area of the square  $ABCD$  is  $(x + y)^2 = x^2 + 2xy + y^2$

Each of the triangles  $\triangle EFA$ ,  $\triangle FGD$ ,  $\triangle GHC$  and  $\triangle HEB$  is a right triangle with height  $x$  and base  $y$ . So, the area of each triangle is  $\frac{1}{2}xy$ . The sum of the areas of the four triangles is

$$(4)\frac{1}{2}xy = 2xy.$$

$EFGH$  is a square with sides of length  $z$ . So the area of  $EFGH$  is  $z^2$ .

Looking at the areas in the diagram we can conclude that:

$$ABCD = \triangle EFA + \triangle FGD + \triangle GHC + \triangle HEB + EFGH$$

On the worksheet, substitute the area expressions (with variables  $x$ ,  $y$ , and  $z$ ) into the equation above and simplify.





5.  $ABCD$  is a square with all sides of length  $(x + y)$ .

The area of the square  $ABCD$  is  $(x + y)^2 = x^2 + 2xy + y^2$

Each of the triangles,  $\triangle EFA$ ,  $\triangle FGD$ ,  $\triangle GHC$  and  $\triangle HEB$ , is a right triangle with height  $x$  and base  $y$ .

So, the area of each triangle is  $\frac{1}{2}xy$ .

$EFGH$  is a square with sides of length  $z$ . So the area of  $EFGH$  is  $z^2$ .

Looking at the areas in the diagram we can conclude that:

$$ABCD = \triangle EFA + \triangle FGD + \triangle GHC + \triangle HEB + EFGH$$

Substitute the area expressions (with variables  $x$ ,  $y$ , and  $z$ ) into the equation above and simplify.

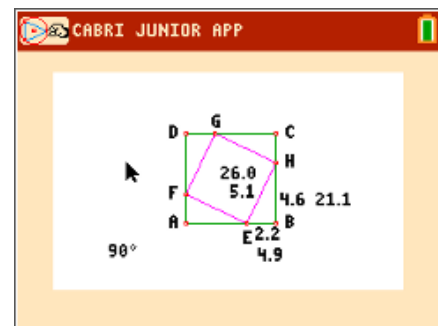
**Answer:**  $ABCD = \square EFA + \square FGD + \square GHC + \square HEB + EFGH$

$$\begin{aligned} &= \frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{2}xy + \frac{1}{2}xy + z^2 \\ &= 2xy + z^2 \\ &= 2xy + x^2 + y^2 \\ &= (x + y)^2 \end{aligned}$$

- Step 11:** Let's look at this numerically as well to confirm what we just proved algebraically. Measure  $\overline{BE}$ ,  $\overline{HB}$ , and  $\overline{EH}$ .

**Note:** Measure  $\overline{HB}$  and  $\overline{EH}$  by pressing **[enter]** on each endpoint because these do not have separate segments constructed.

Use the **Calculate** tool to find the squares of these lengths.



- Step 12:** Find the sum of the squares of the lengths of  $\overline{BE}$  and  $\overline{HB}$ .

In the right triangle  $HBE$ , is  $BE^2 + HB^2 = EH^2$ ?

Drag point  $E$  to ensure that the relationship holds for other locations of the points  $E$ ,  $F$ ,  $G$ , and  $H$ .

What would happen if you dragged one of the points  $A$  or  $B$ ? Would the relationship still hold?



6. Record three sets of numeric values for  $\triangle HEB$ .

**Sample Answers:**

$BE$	$BE^2$	$HB$	$HB^2$	$BE^2 + HB^2$	$EH$	$EH^2$
2.2	4.9	4.6	21.1	26.0	5.1	26.0
1.1	1.2	2.3	5.3	6.5	2.5	6.5
1.9	3.7	4.0	15.9	19.6	4.4	19.6

7. Does  $BE^2 + HB^2 = EH^2$  when  $E$  is dragged to a different location?

**Answer:** yes

8. Does  $BE^2 + HB^2 = EH^2$  when  $A$  or  $B$  is dragged to a different location?

**Answer:** yes