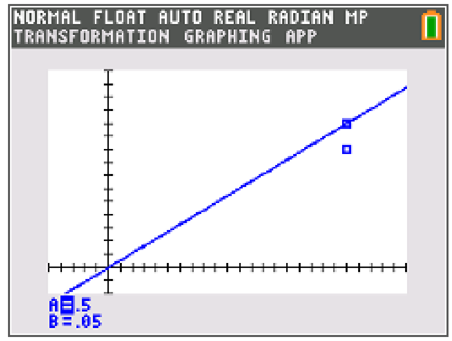
Boats in Motion



About the Lesson

In this activity, students will explore the motion of a boat going up and down the river. They will be instructed to solve the resulting system of equations algebraically and graphically.
As a result, students will:

- Use the transformation graphing application to explore the slope of a distance-time graph.

Vocabulary

- system of equations

Teacher Preparation and Notes

- The student worksheet provides instructions and question to guide the inquiry and focus the observations.
- The Transformation Graphing App is used in this activity.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

* *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Boats_in_Motion_Student.pdf
- Boats_in_Motion_Student.doc

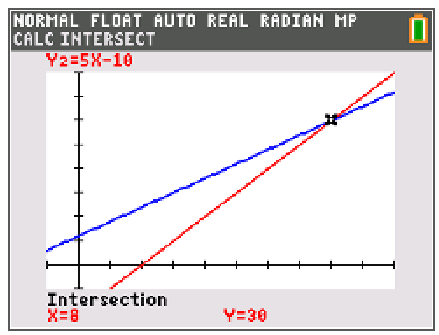
Problem 1 – Boat Motion & Graphically Solve

Introduce the problem of finding the distance between two towns across a lake. The distance between the towns can be determined using the rate of a boat traveling between the towns on still water and the time it takes to travel.

Once students understand the goal of using the rate of the boat to determine distance, ask them how the situation changes if, instead of across a lake, the towns are along a river and the boat has to travel with the current and against the current to travel between towns.

Teacher Tip: Ask students why the boat goes faster with the current than against the current. They should know that the boat goes faster with the current making the rate relative to the shore increase, while it has to fight the current when going in the opposite direction, making the rate decrease. Conclude by asking the students how far they would go in 4 hours traveling with the current, and how far they would go in 7 hours traveling against the current.

For the purposes of this activity, we assume that the boat is traveling parallel to the current, that the boat takes the exact same path when it travels with the current and against the current, and that both the current and boat have constant speed.



1. Let *r* be the rate of the boat in still water. How could the rate with the current and the rate against the current be expressed?

Answer: with current: $r + 2$; against current: $r - 2$

2. Use the above information to fill in the blank spaces.
distance = rate × time

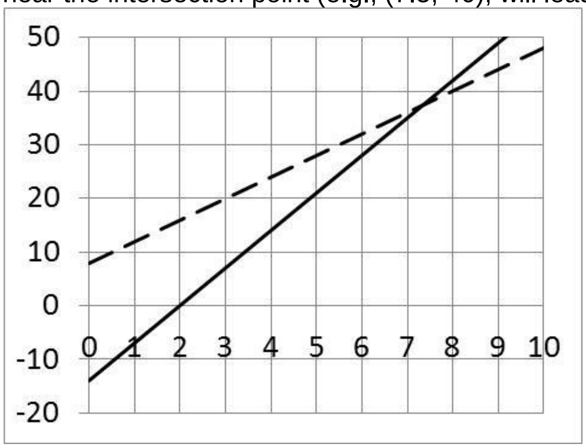
Answer: with current: $d = (r + 2) \times 4$; against current: $d = (r - 2) \times 7$.

3. Set the equations equal to each other and solve for r algebraically. Show your work here.

Answer:

$$4r + 8 = 7r - 14$$
$$3r = 22$$
$$r = 7.\overline{333} \text{ mph}$$
$$r = 7\frac{1}{3} \text{ mph}$$

Solving the equation graphically, one plots the lines $y = 4x + 8$ and $y = 7r – 14$. Selecting any point near the intersection point (e.g., (7.5, 40), will lead to the intersection point (7.3333333, 37.333333).



4. How does this point compare with your solution from Question 3?

Answer: Both the algebraic and graphical solutions lead to the same answer.

Record the rate of the boat in still water and the distance between the towns. Express the rate to the nearest tenth of a mph and the distance to the nearest mile. Include units.

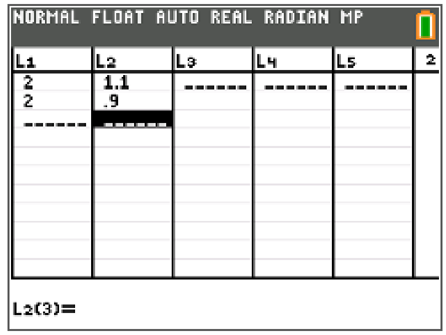
Sample Answer: To the nearest tenth of a mph, the rate is 7.3 mph. Substituting this into either equation and expressing the result to the nearest mile gives a distance of 37 miles. Alton and Barnhart are about 37 miles apart along the Mississippi in the St. Louis area.

Problem 2 – Distance-Time Graph, Explore Slopes

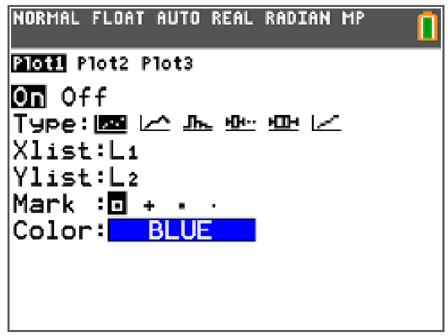
Using $d = rt$ for this situation gives the following equations:

$$1.1 = (A + B)\ 2$$
$$0.9 = (A - B)\ 2$$

where A is the rate (speed) of the steam engine and B is the rate (velocity) of Velma's walking.

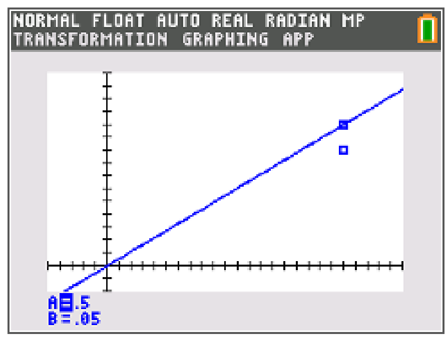


Students should set up lists L_1 and L_2 and the scatter plot settings as shown at right.



On the graph, students are to use the arrows keys to change the rates of the train (A) and Velma's walking (B) so that the lines go through the points (2, 1.1) and (2, 0.9). The slopes of the lines represent the rates of A and B. Using the **Transformation Graphing** App, students will only be able to graph one equation at a time, but they should see that once one equation has been graphed correctly, the other will be correct as well (no need to change the values of A or B).

To solve this system of equations, students are to distribute and then add the first equation to the second. The solution is the train moves at 0.5 mi/min and Velma walks 0.05 mi/min.



5. What does the slope of the line in the distance-time graph represent?

<u>**Sample Answer:**</u> the sum of the speed of the steam engine and the speed of Velma's walking and the difference of the speed of the steam engine and Velma's walking

6. Apply $d = rt$ to this situation. What does r equal?

<u>**Answer:**</u> forward: $1.1 = (A + B) \times 2$; back: $0.9 = (A - B) \times 2$

7. Algebraically solve the equation.

Answer: $1.1 = (A+B)\times 2 \rightarrow 0 = (A+B)\times 2 - 1.1;\ 0.9 = (A-B)\times 2 \rightarrow 0 = (A-B)\times 2 - 0.9$

$$\begin{aligned}(A+B)\times 2 - 1.1 &= (A-B)\times 2 - 0.9\\ 2A+2B-1.1 &= 2A-2B-0.9\\ 2B-1.1 &= -2B-0.9\\ 4B-1.1 &= -0.9\\ 4B &= 0.2\\ B &= 0.05\text{ mi/min}\end{aligned}$$

$$\begin{aligned}1.1 &= (A+B)\times 2\\ 1.1 &= (A+0.05)\times 2\\ 0.55 &= A+0.05\\ 0.5\text{ mi/min} &= A\end{aligned}$$

Problem 3 – Extension/Homework

8. An airplane flew 3 hours with a tail wind of 20km/h. The return flight with the same wind took 3.5 hours. Find the speed of the airplane in still air.

Answer: west: $D = (r + 20)\ 3$; east: $D = (r - 20)\ 3.5$

$$\begin{aligned}3r+60 &= 3.5r-70\\ 0.5r &= 130\\ r &= 260\text{ km/h}\end{aligned}$$

The speed of the airplane in still air is 260 km/h.

9. A car is driving along a straight road at 50 feet per second. As it passes a second car sitting at rest, the second car accelerates, traveling a distance in time *t* equal to $d = 4.5t^2$, where *d* is in feet and *t* is in seconds. Determine the time it takes for the second car to catch up to the first car, and the distance it has traveled in that time. Express the time to the nearest tenth of a second and the distance to the nearest foot. Explain the steps used for both an algebraic solution and a graphical solution using the calculator.

Algebraic Solution:

1. Write an equation for the distance the first car travels: $d = 50t$.
2. Write an equation for the distance the second car travels: $d = 4.5t^2$.
3. Set the equations equal to one another.

   $50t = 4.5t^2$
4. Solve for *t*.

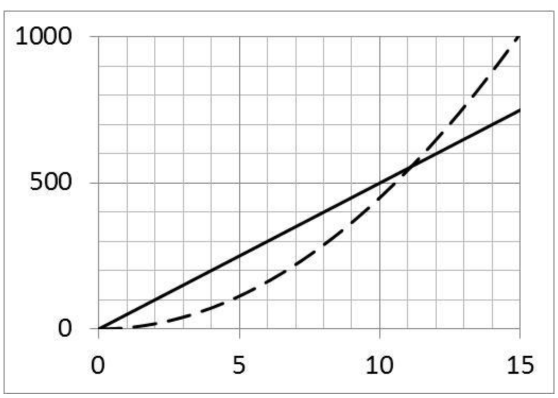
$$\begin{aligned} 50t &= 4.5t^2 \\ \frac{50t}{4.5t} &= \frac{4.5t^2}{4.5t} \\ 11.1 &\approx t \\ t &\approx 11.1 \text{ seconds} \end{aligned}$$

5. Solve for *d*.

$$\begin{aligned} d &= 50t \\ &= (50)(11.1) \\ &= 555 \text{ feet} \end{aligned}$$

Graphical Solution:

Plotting the two equations and finding the intersection point produces the same results.



Discussion of the two procedures will be somewhat open-ended. Urge comparison of the relative difficulty of the two, identifying situations where the algebraic might be simpler (examples such as these are easily treated algebraically), and situations where the graphical might be simpler (cases where algebraic solutions may not even exist, e.g., equations where polynomials and exponentials are both involved).

