

Math Objectives

- Students will explore the validation of a model constructed from a data set.
- Students will be able to determine a regression equation.
- Students will understand that a pattern in the graph of the residuals indicates that the model is not appropriate for the data set.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

• data set

linear

residual

exponential

About the Lesson

- This lesson includes modeling a data set using linear and exponential regressions.
- As a result students will:
 - Observe a residual plot as well as a scatter plot of the data.
 - Understand that a residual is the actual value of the data minus the predicted value from the regression equation.
 - Conjecture and draw conclusions about the appropriateness of the model based on the residual plot.

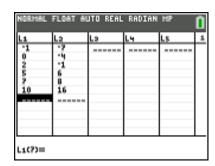
Teacher Preparation and Notes.

• This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

 Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

 * with the latest operating system (2.55MP) featuring MathPrint TM functionality.



Tech Tips:

- This activity includes screen
 captures taken from the TI84 Plus CE. It is also
 appropriate for use with the
 rest of the TI-84 Plus family.
 Slight variations to these
 directions may be required if
 using other calculator
 models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <u>http://education.ti.com/calcul</u> <u>ators/pd/US/Online-</u> <u>Learning/Tutorials</u>

Lesson Files:

Student Activity Residuals_84_Student.pdf Residuals_84_Student.doc



In this activity, you will investigate a residual plot for a set of data after selecting a regression model. The residual plot is used to justify the choice of a function model based on an analysis of the residuals.

NORMAL	FLOAT AL	JTO REAL	RADIAN	HP	Û
1 0 2 5 7 10	L3 -7 -4 -1 6 8 16 			<u>Ls</u>	1
L1(?)=					

Discussion Points and Possible Answers

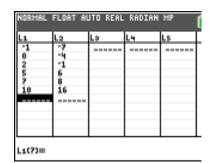
Tech Tip: If data exists in the lists of the Stats Editor, advise students to press the up arrow until L1 (or any other lists) is highlighted. Select clear and enter to delete the data. An alternate method is given in Part 2.

Part 1

Use the following data set in Part 1.

x	-1	0	2	5	7	10
у	-7	-4	-1	6	8	16

1. To enter the data, select stat 1: Edit... Enter the *x* values in [L1] and the *y* values in [L2].



Tech Tip: Students must press enter or the down arrow after the last value is entered. If the students gets a Dimension Mismatch error, they should check their data to make sure all the data has been entered.

TEACHER NOTES



To run a linear regression, select stat and use the right arrow to highlight CALC.

Select 4: LinReg (ax + b). Make sure the Xlist: is set to L1 and the Ylist: is set to L2. Arrow down to Store RegEQ: and press alpha trace to select 1: Y1. Arrow down to Calculate and press ENTER. The linear regression is calculated and is also stored in Y1. What is your linear regression equation?

<u>Answer:</u> The linear regression equation is y = ax + b a = 2.003669725b = -4.680733945

 To view the scatter plot, press 2nd Y= to access STAT PLOTS. Select 1: Plot 1 and press ENTER. Use the arrow keys to change the settings to match the screen to the right. Select (200M) 9: ZoomStat.

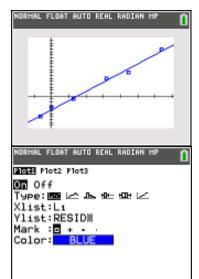
Note: To hide the graph of the linear regression equation, select

 $Y_{=}$, use the left arrow key to place it on the = sign and press enter. Select graph to view the scatter plot.

The residual is the actual value minus the predicated value. A regression model is justified as appropriate for a data set if the residuals of a regression, the residual plot, appear without pattern. To view the residual plot, press 2nd Y= and edit the settings of Plot 1 to match the screen to the right. Note: Resid is found by pressing 2nd stat. Select Z00M 9: ZoomStat.



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On Off	
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Ylist:L2	
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TEACHER NOTES



Does your residual plot have a pattern? Would a linear regression be appropriate for this data set?

<u>Answer:</u> The residual plot does not have a pattern. A linear regression is appropriate for this data set.

3. To evaluate the predicted values, press alpha trace to select 1: Y1. Calculate Y1(-1) and then calculate the residual when x is -1. Calculate Y1(0) and then calculate the residual when x is 0. Notice that one residual value is negative and one is positive. What does this tell us about the predicted value as being an underestimate or an overestimate?

Answer:

Y1(-1) = -6.68440367; Residual = -0.31559633Y1(0) = -4.680733945; Residual = 0.680733945If the residual is positive, the predicted value is an underestimate. If the residual is negative, the predicted value is an overestimate.

Note: To view the residual list for all of the data points, select stat 1: Edit... Arrow to the right until you get to L6. Press the Up arrow and then the right arrow. Open the List Editor by selecting 2nd stat. Select 7: RESID and press ENTER.

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-76.68440367 -0.3155 Y1(0)	59633
Y1(0)	2004E
-4.680733945	13X45.
0.68073	3945
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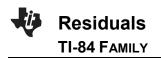
La	L4	Ls	Lő	RESID
				-0.316 0.6807 -0.327 0.6624 -1.345 0.644

Teacher Note: The graph of the scatter plot with the regression equation may be helpful to visualize the overestimate or underestimate.

Part 2

Use the following data set in Part 2.

x	-1	0	1	2	4	5
у	0.2	0.6	0.9	2.1	7.9	16.2



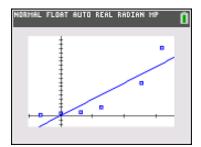
Tech Tip: To clear the data and residuals from the previous problem, select **stat** and 4: ClrList, then add L1, L2, LRESID separated by commas.

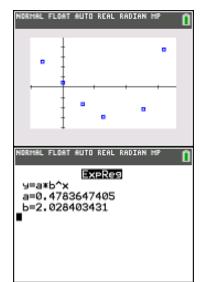
4. Follow the steps in Part 1. Enter the data in L1 and L2. Compute a linear regression, view the scatter plot, and view the residual plot. Does your residual plot have a pattern? Would a linear regression be appropriate for this data set?

<u>Answer:</u> The residual plot appears to have a pattern. A linear regression is not appropriate for this data set.

L1	La	La	L4	Ls	L
-1 0 1 2 4 5	0.2 0.6 0.9 2.1 7.8 16.2				
					L



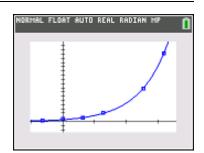


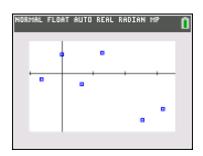


5. Now compute an exponential regression which is 0: ExpReg in the Stats Calc menu. View the scatter plot, and the residual plot. Does your residual plot have a pattern? Would an exponential regression be appropriate for this data set?



Answer: The residual plot does not appear to have a pattern. An exponential regression is appropriate for this data set.





Extensions

1. Find a data set that models a quadratic and ask the students to follow the steps for a quadratic regression. Observe the scatter plot and the residual plot.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to input data in the Stats Editor.
- How to compute linear, quadratic, and exponential regressions.
- How to plot residuals and determine if the residual plot has a pattern.