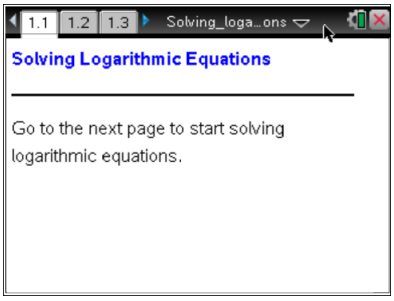
Solving Logarithmic Equations

Student Activity

Name _____

Class _____

Open the TI-Nspire document
Solving_Logarithmic_Equations.tns.



We know that $\log_3 3 = 1$ and $\log_3 9 = 2$, but is there an approximate solution to the equation $\log_3 x = 1.5$? An exact solution? In this activity, you will explore the answer to these questions numerically, graphically, and algebraically.

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1. Estimate the solution to the equation $\log_3 x = 1.5$ using the following numeric pattern.

$\log_3 3 = 1$
$\log_3 x = 1.5$
$\log_3 9 = 2$

Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

2. The table shows inputs and outputs for the function $f(x) = \log_3 x$.
   a. Input your estimate from question 1 into cell A2. Input other values to get the output as close as possible to 1.5. Record your closest input and output below:

$f(3) = 1$
$f(\_\_\_\_\_) = \_\_\_\_\_$
$f(9) = 2$

   b. Is there an input value that results in an output value of exactly 1.5?

Move to page 1.3.

3. The graph of the function $f(x) = \log_3 x$ is shown along with its inverse $f^{-1}(x) = 3^x$. Point P' is the reflection of point P over the line $y = x$.
   a. Suppose the coordinates of P are $(3, 1)$. Write an exponential equation by substituting the coordinates of P' into the function $f^{-1}(x) = 3^x$.

b. Move point P' so that the input of the function $f^{-1}(x) = 3^x$ is 1.5. According to the graph, what is the *approximate* solution to the equation $\log_3 x = 1.5$? Why is this an approximate solution?

c. Recall that the composition of any function and its inverse always results in the *x*. In other words, $f \circ f^{-1}(x) = f\left(f^{-1}(x)\right) = x$. As such, the composition of $f(x) = \log_3 x$ and $f^{-1}(x) = 3^x$ results in the equation $\log_3 3^x = x$. Use this composition relationship to find the *exact* solution to the equation $\log_3 x = 1.5$. What is the exact solution?

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4. Solve the equation $\log_3 x = 1.5$ by changing the base and reducing the left side of the equation to *x*. To change the base, click the up and down arrows.
   a. What base results in the exact solution?

   b. What is the exact solution to $\log_3 x = 1.5$? Why?

Move to page 1.5.

5. You found an approximate numeric solution in questions 1 and 2, an exact graphical solution in question 3, and an exact numerical solution in question 4. Compare your solutions using the Calculator page provided. How do your solutions compare?

6. Estimate the solution to the equation $\log_2 x = 3.2$ using the following numeric pattern:

$\log_2 8 = 3$
$\log_2 x = 3.2$
$\log_2 16 = 4$

Move to page 2.1.

7. The table shows inputs and outputs for the function $f(x) = \log_2 x$. Input your estimate from question 6 into cell A2. Input other values to get the output as close as possible to 3.2. Record your closest input and output below:

$$f(8) = 3$$
$$f(\_\_\_\_\_) = \_\_\_\_\_$$
$$f(16) = 4$$

Move to page 2.2.

8. The graph of $f(x) = \log_2 x$ is shown along with its inverse $f^{-1}(x) = 2^x$. Point P' is the reflection of point P over the line $y = x$. Move point P' so that the input of $f^{-1}(x) = 2^x$ is 3.2. According to the graph, what is the *approximate* solution to the equation $\log_2 x = 3.2$?

Move to page 2.3.

9. Solve the equation $\log_2 x = 3.2$ by changing the base and reducing the left side of the equation to *x*. To change the base, click the up and down arrows.
   a. What base results in the exact solution?

   b. What is the exact solution to $\log_2 x = 3.2$? Why?

Move to page 2.4.

10. Determine how close your estimates from questions 7 and 8 were by entering your exact answer from question 9 in this Calculator page. How do your solutions compare?

11. Use the algebraic methods from questions 4 and 9 to find exact solutions to these equations.

a. $\log_5 x = 1.3$

b. $\log_7 x = \sqrt{2}$

c. $\log_6 x = \frac{-10}{9}$