Problem 1 – Negative Angle Identities

Graph sin(*x*) and sin(–*x*) together. Estimate the horizontal "difference" between the two curves by noting the *x*-values of their peaks.

- sin(–*x*) has a peak at *x* = _____.
- sin(*x*) has a peak at *x* = _____.

Translate sin(*x*) to the left or right until it aligns with sin(–*x*). What is the new equation?

- sin(–*x*) = ________________

Complete the geometric proof of this negative angle identity.

Proving the Negative Angle Identities

In Cabri Jr. open the file named **NEGANGLE**.

1. Reflect segment *R* over the *x*-axis. Label the point where the reflected segment intersect the circle **P"**. Find the coordinates of point *P*" and *P*.
2. Use the coordinates of point *P*" to write an expression for sin(–*T*). The angle formed by the *x*-axis and the reflected segment is –T.
3. Substitute $\sin(T) = \frac{y}{r}$ in the expression to get sin(–*T*) = –sin(*T*), the negative angle identity you found in the graph! (If you replace *T* with x).
4. Repeat these steps to find expressions for cos(–*x*) in terms of cos(*x*) and tan(–*x*) in terms of tan(*x*).

   cos(–*x*) =   tan(–*x*) =

Verify the negative angle identities by graphing.

Problem 2 – Cofunction Identities

- Enter **sin(X)** in **Y1** and **cos(X)** in **Y2**. How do the graphs relate?
- How are the graphs of sin(*x*) and cos(*x*) the same? How are they different? How can you translate the graph of Y2 to make it line up with Y1?
- Estimate the horizontal "difference" between the two curves by noting the x-values of their peaks.
- sin(*x*) has a peak at *x* = _____.
- cos(*x*) has a peak at *x* = _____.

Use what you know about translating graphs to change the equation of cos(*x*) to shift it to the left or right until it aligns with the graph of sin(*x*).

- sin(*x*) = cos (_______)

Complete the geometric proof of this cofunction identity. Open the file **COFUNCT** in Cabri Jr.

- Measure angles *S* and *T*. How are the two acute angles in a right triangle related? Use your answer to write an expression for *S* in terms of *T*.   *S* = ____________

Proving the Cofunction Angle Identities

1. Use the definition of sine as opposite/hypotenuse to write an expression for the sin(S). Substitute 90 – *T* for *S* and cos(*T*) for $\frac{A}{C}$ to get sin(*T*) = cos (90 – *T*).

2. Substitute *x* for *T* and change degrees to radians to get $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$.

3. Use the negative angle identity to rewrite $\cos\left(\frac{\pi}{2} - x\right)$ as $\cos\left(-\left(\frac{\pi}{2} - x\right)\right) = \cos\left(x - \frac{\pi}{2}\right)$.

4. Repeat steps 1 and 2 above to write expressions for cos(x) and tan(x).

- cos(*x*) = tan(*x*) =

Verify the cofunction identities by graphing.

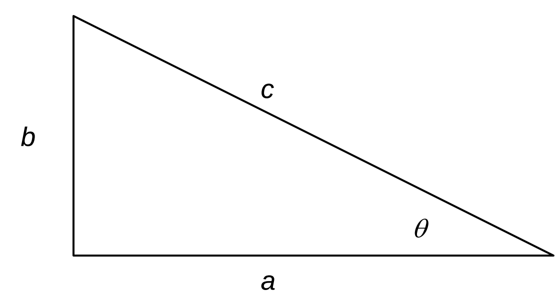
Problem 3 – A closer look at amplitude, period, and frequency

Enter **$(\sin(X))^2$** in **Y1** and **$(\cos(X))^2$** in **Y2**. Use what you know about translating graphs to change the equation in $\cos^2(x)$ to flip it and then shift it vertically to make it align with the graph of $\sin^2(x)$.

- $\sin^2(x)$ = ___________________

Proving the Pythagorean Angle Identities

Use the diagram and follow these steps to prove the Pythagorean identities.



1. Write the Pythagorean Formula: $a^2 + b^2 = c^2$.

2. Divide both sides of the Pythagorean Formula by c^2: $\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$

3. Simplify the result. Substitute sin θ for $\frac{b}{c}$ and cos θ for $\frac{a}{c}$ to yield $\sin^2(x) + \cos^2(x) = 1$.

4. Repeat steps 1 through 3, dividing by a^2 and b^2 to yield additional identities.

- $\sin^2(x) + \cos^2(x) = 1$
- 1 + _____ = ______
- $\tan^2(x)$ = _____ – 1

Verify the Pythagorean identities by graphing.