



Math Objectives

- Students will identify a situation involving binomial trials.
- Students will observe that a binomial distribution is a function of both sample size and the probability of a success.
- Students will interpret a table of binomial probabilities.
- Student will answer probability questions using the graph of a binomial model for a random variable involving binomial trials.

Vocabulary

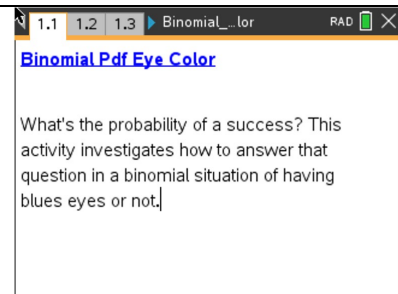
- binomial distribution
- expected value
- probability

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 4 Statistics and Probability:
 - 4.8 Binomial Distribution as a model and found using technology. Mean and variance of a binomial distribution.

This lesson also involves binomial trials, distributions, and probabilities. Students can create the tns file following the steps in Binomial_Pdf_Create_Eye_Color, or they can use the premade file Binomial_Pdf_Eye_Color.tns

- As a result, students will:
 - For a binomial distribution with a given sample size and probability of success
 - find the probability of specific outcomes from a spreadsheet of values.
 - find the probability of specific outcomes from the graph.
 - Find the probability of mutually exclusive events from a graph or spreadsheet.
 - Change the sample size and the probability of a success and find the probability of specific outcomes.



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity

Binomial_Pdf_Eye_Color_Student.pdf

Binomial_Pdf_Eye_Color_Student.doc

Create File

Binomial_Pdf_Eye_Color_Create.doc

TI-Nspire document

Binomial_Pdf_Eye_Color.tns



- Analyze how the probabilities change as the sample size increases and as the probability of a success increases.


TI-Nspire™ Navigator™ System

- Use Screen Capture to compare graphs of the binomial distributions as the sample size and the probability of a success change.
- Use Quick Poll to check for student understanding.
- Use Live Presenter to demonstrate how to create sliders.

Prerequisite Knowledge

- Students should be familiar with the concept of a binomial distribution.




- Notation and Terminology to be Familiar with...*

In a sequence of n independent trials of an experiment in which there are exactly two outcomes “success” and “failure” with constant probabilities $P(\text{success}) = p$, $P(\text{failure}) = 1 - p$, if X denotes the discrete random variable equal to the number of successes in n trials, then the probability distribution function of X is

$$P(X = x) = C_x^n \cdot p^x \cdot (1 - p)^{n-x}, x \in \{0, 1, 2, \dots, n\}$$

These facts are summarized with the words “ X is distributed binomially with parameters n and p ” and with symbols $X \sim B(n, p)$.

Activity Materials

Compatible TI Technologies:  TI-Nspire™ CX Handhelds,
 TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software

Discussion Points and Possible Answers

Tech Tip: Moving the cursor over a bin in a histogram will display the values in the interval spanned by the bin and the probability of those values; [18.5,19.5) 0.0558 indicates about 6% of all 50 trial experiments will result in exactly 19 successes.



Several sources indicate that the probability of a person in the United States having blue eyes is approximately 30%. Suppose you randomly sampled 50 people in the United States. One basic question to answer is whether an underlying probability model might describe the probability of the possible numbers of blue-eyed people in your sample.

1. This situation involves binomial trials, so the first step is to check whether the requirements for binomial trials are met: two outcomes per trial; a fixed number of trials; a constant probability of success; and independent trials. Verify that these conditions are met in this context.

Teacher Tip: A binomial distribution is sometimes related to Bernoulli trials— according to some interpretations, a situation involving Bernoulli trials has the same requirements as binomial trials.

Answer: The problem is formulated with two outcomes and exactly 50 trials. A success is a person with blue-eyes. Any other eye color would be classified as not a success. The probability of being blue-eyed is assumed to be constant for the population based on a variety of sources. Under the assumption that the random sample of people has been selected independently, the conditions for binomial trials are met.

2. a. Reflect on how you built the .tns file. State two inputs that determined the binomial distribution.

Answer: The probability 0.3 and the sample size, n .

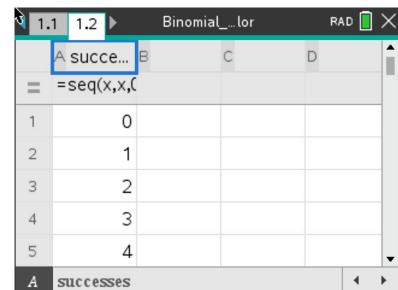
- b. For a random sample of 50 people, state what would you expect to be the most likely number of blue-eyed people. Explain your reasoning.

Answer: The most likely number of blue-eyed people will probably be the expected value (mean), which is $(0.3)(50)$ or 15.

Move to page 1.2.

3. In terms of the context,
 - a. state what the zero represents in cell A1.

Answer: The zero indicates the outcome of having no blue-eyed people in the sample of 50 people.





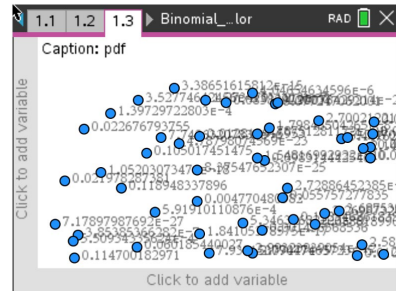
- b. state what the value in cell B5 represents.

Answer: The value is 0.00014, the probability of getting exactly 4 blue-eyed people in the sample of 50 people.

Move to page 1.3.

If the conditions for binomial trials are met, a binomial model can be used to represent the probabilities, which is the distribution you created on Page 1.3.

4. a. Describe the distribution in terms of shape, center, and spread.



Answer: The distribution is mound shaped but not quite symmetric. The center seems to be at 15, and the most likely outcomes seem to be between 10 and 21.

Teacher Tip: You might ask students how their answer to 2b relates to the graph on Page 1.3. Some important points to discuss include: What are the end points of the graph? How is this distribution similar to and different from a normal distribution? This would help students recognize that the probabilities of values far from 15 do exist but are very small and that the binomial distribution is a discrete distribution while the normal curve is continuous. Here you cannot have "0.5" success.

- b. Describe how the context of the eye color for a sample of randomly chosen people relates to the graph.

Answer: A success is being blue-eyed, and the probability of a success is given as 0.30. Each bin represents the probability of having that number of blue-eyed people in a sample of 50 randomly-chosen people.

- c. Sandra wonders whether the probability of getting 4 blue-eyed people in the sample of 50 was 0. State what you would tell her.

Sample Answers: One response would be to return to the table on Page 1.2 and look for the probability associated with 30 blue-eyed people in the sample, 0.000008. The graph on Page 1.3 does not show the results for such very small probabilities. Some students might calculate the value using the binomial distribution formula.



5. Suppose x represents the number of blue-eyed people. Describe how the graph helps you answer each of the following:
- The most likely outcomes.

Sample Answers: Some students might suggest the outcomes represented by the tallest bin, or 15: $P(x = 15) = 0.1223$. Others might combine the tallest bars, for example from 14 to 16 blue-eyed people in the sample of 50; $P(14 \leq x < 16) = 0.1147 + 0.1223 + 0.1189 = 0.3559$.

- The least likely outcomes.

Sample Answers: Depending on how students think about "least likely," the answers will vary. Some students might use the outcomes in the tails, $P(x = 7) = 0.0048$ and $P(x = 23) = 0.0067$. Others might include the outcomes whose probabilities are so small they did not show up in the distribution, any outcome less than 7 or greater than 23; $P(x < 7 \text{ or } x > 23)$.

- If you add the probabilities of every possible outcome, state what the result should be.

Answer: 1 because the probabilities of all possible outcomes sum to 1.

6. a. State the approximate probability that exactly 10 people will have blue eyes, $P(x = 10)$.

Answer: Approximately, 0.0386.

- State the approximate probability that 10 or 11 people will have blue eyes, $P(x = 10, 11)$.

Answer: The sum of the probability of each outcome, 10 and 11, will be approximately $0.0386 + 0.0602 = 0.0988$.

Teacher Tip: Students should be aware that you can add the probabilities only because the events are mutually exclusive. You might ask them why this is important in a class discussion and could refer to a Venn diagram for events that are not mutually exclusive (the probability of a blonde haired person having blue eyes, for example).

- State the approximate probability that fewer than 10 people will have blue eyes, $P(x < 10)$.

Answer: $P(0 \leq x \leq 9)$ where $P(0 \leq x \leq 6)$ is very small, so approximately $0 + 0.0048 + 0.0220 + 0.0110 = 0.0378$.



- d. State the approximate probability that at least 10 people will have blue eyes, $P(x \geq 10)$. Describe how could you use your answer to part **b** to find the answer to this question.

Answer: You can subtract the probability for $P(x < 10)$ from 1, the sum of all the probabilities; $P(x \geq 10)$ is approximately $1 - 0.0378 = 0.9622$.

7. Suppose a researcher was checking the number of candies of a certain color chosen from randomly selected bags where each bag has exactly 50 candies. The manufacturer claimed that the probability of getting a green candy was approximately 0.3.

- a. Describe how this context is alike or different from the context involving eye color above.

Answer: While the context is different, the two parameters for a binomial distribution are the same: $n = 50$ and $p = 0.3$.

- b. In a sample size of 50 candy pieces, if the researcher found the number of green candies in the sample indicated below, explain if he should be surprised or not.

- i. 12

Sample Answers: No because the chance that this would occur is 10.5%, which is not too unusual. Other students might think this is a likely event.

Teacher Tip: Deciding what is "unlikely" can provide an opportunity to consider different benchmarks and raise the question: where would you begin to think an outcome was not due to chance – 1 out of 4; 1 out of 10; 1 out of 20? The discussion can lay the foundation for later work with confidence levels.

- ii. 8

Answer: Yes because this would occur randomly only 1% of the time.

- c. If the researcher were surprised, state the conclusions he might draw.

Sample Answers: The outcome might just be due to chance, or it could be that the manufacturer's claim about the number of candies was suspect.

Teacher Tip: This is an informal approach to inference and hypothesis testing.


Extension

Adding the two sliders to the .tns file as described in the Extension instructions of the Binomial Pdf Eye Color Create document allows you to change both the sample size and the probability of a success.

Teacher Tip: You might want students to compare distributions with the same sample size for complementary probabilities (i.e., 0.05 and 0.95; 0.10 and 0.90).

Teacher Tip: You might want to relate the formula for finding $p(x)$ where x is a random variable in a binomial distribution to the outcomes visible in the graph to help students explore how changing the values of n and p affect the probabilities.

Tech Tip: This lesson is set up to focus primarily on interpreting the graphical representation of the binomial distribution; but if you would like students to compare the table and the graph, you can do so by altering Page 1.2. On Page 1.2, select doc > **Page Layout** > **Select Layout** > **Layout 2**. Then follow the steps in the Binomial Pdf Eye Color Create document for creating the graph and the sliders.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

8. The probability of having green or light brown eyes with green specks is about 12%. In a random sample of 100 people, find the probability of the following:
- a. Exactly 6 people with that eye color.

Answer: Approximately, 0.0215

- b. At least 16 people with that eye color.

Answer: Approximately, $P(x \geq 16) \approx 0.141$

- c. Anywhere from 7 to 15, including 7 and 15, people with that eye color.

Answer: Approximately, $P(7 \leq x \leq 15) \approx 0.822$



9. a. Before using the arrows to change anything, predict how your answers to question 8 might change if the sample size were 30 people.

Answer: Students might suspect that the probability of getting at least 16 blue-eyed people will decrease considerably because the expected value is $(0.12)(30)=3.6$ or about 4, and 16 seems to be farther in the right tail of the distribution than it would be when the expected value is 12.

- b. Check your prediction by changing the sample size to 30 and using the resulting distribution.

Answer: The probability of exactly 6 would be approximately 0.0825 or 8%; an outcome at least 16 is very close to 0; from 7 to 15 inclusive is about 6%.

10. a. Predict how your answers to question 7 would change if the sample size remained 100 but the probability of brown eyes was 0.40 or 40%.

Answer: The distribution will shift and center around 40, so outcomes below 20 (including two asked for in problem 7, exactly 6 and from 7 to 15) and above 55 will be very unlikely. The probability of an outcome at least 16 is nearly certain.

- b. Check your prediction by changing n and p accordingly and using the resulting distribution.

Answer: The probability of exactly 6 would be close to 0; at least 16 is about 99.999%; from 7 to 15 inclusive is close to 0.

11. a. Describe how the distribution changes as the sample size goes from 100 to 50 people for $p = 0.4$.

Answer: The center shifts from 40 to 20, and the range of the probabilities not close to zero decreases from about 30 to about 20, which means the distribution has a higher peak for the smaller n .

- b. State how your answer to part **a** would change if the probability of a success was a different value.

Answer: The center would shift to the expected value, but in general, as the sample size decreases, the distribution becomes tighter.

- c. Fix the sample size at $n = 50$. Change the values of p , and describe what happens.



Answer: For very small p -values, the distribution is skewed right and becomes symmetric. For very large p -values, the distribution is skewed left.

TI-Nspire Navigator Opportunity: Quick Poll

See Note 2 at the end of this lesson.

Tech Tip: If you use a large n and a large p , you might need to select **Menu > Window/Zoom > Window Settings...** and change the **x Max** to a larger value.

Teacher Tip: Students might need to further investigate what happens to the probabilities of an outcome as the sample size increases in general and what happens to the probability of an outcome if the sample size remains constant and the probability of a success decreases. This should lead them to notice that for small probabilities the distributions are not symmetric. The Statistics Nspired activity "Why np min" could be a useful resource.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- A binomial distribution is determined by the sample size and the probability of a success.
- How to find the probability of mutually exclusive events by adding the probabilities of each event or by using the fact that the probability of all the outcomes is 1.
- How to use a graph of the binomial distribution to find the probability of an outcome for a given sample size and given probability of a success.

Assessment

Select the probability of a success and a sample size. Make up three questions and find the answers – exchange questions with your partner, then check each other's answers.

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Note 1

Live Presenter

Live Presenter might be used in the extension to have a student demonstrate how to build the sliders in the .tns file.

Note 2

Question 11, Quick Poll

A Quick Poll can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students may have.



***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*