

Volume of a Watermelon

Student Activity

7 8 9 10 11 12



TI-84PlusCE™



Activity



Student

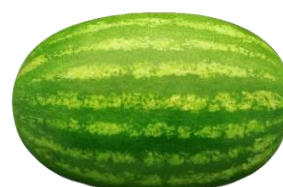


50 min

Introduction

This activity looks at three different ways to measure the volume of a watermelon.

- Archimedes' principle (measurement by displacement of water)
- Approximation – Cutting the watermelon into slices that can be approximated by cylinders
- Calculus – Determining an equation to model the shape of the watermelon and rotating it around an axis.



Part 1 - Archimedes

For this measurement you will need:

- Watermelon
- Bucket – big enough to submerge your watermelon*.
- Measuring jug
- Water

*If you don't have a bucket big enough, try one that will hold half your watermelon. Archimedes' principle states:

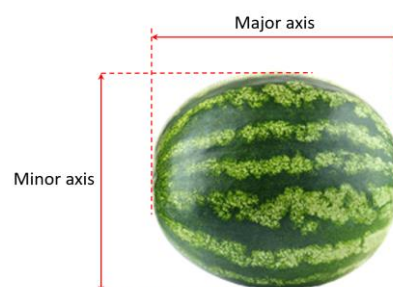
A body at rest in a fluid is acted upon by a force pushing upward called the buoyant force, which is equal to the weight of the fluid that the body displaces. **If the body is completely submerged, the volume of fluid displaced is equal to the volume of the body.**

Completely fill the bucket with water. Place the watermelon in the bucket, you will most likely need to nudge the watermelon to ensure it is completely submerged, avoid displacing too much water with your hand. Once the water has settled, remove the watermelon from the bucket, again, try and avoid displacing any additional water. Once the watermelon is removed from the bucket, use your measuring jug to refill the bucket and therefore determine the volume of water that was displaced by the watermelon.

Question: 1.

According to your measurements and Archimedes' principle, what is the volume of your watermelon.

Note: 1 litre = 1000cm³



Part 2 – Preparing your watermelon

For this section you will need:

- Watermelon (same)
- Camera / phone
- Computer + TI-Connect™ CE software + USB cable
- Ruler / Tape measure

It is important that the image be an accurate scaled version of the actual watermelon. Measure and record the major and minor axis lengths carefully.

Step 1:

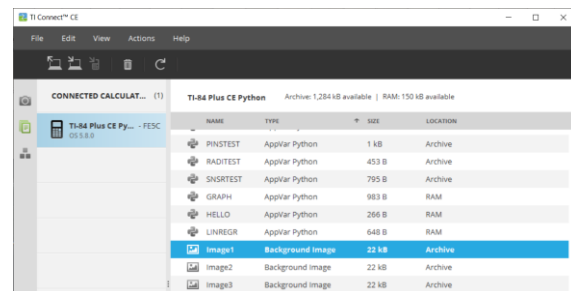
Photograph your watermelon. Make sure the image is 'square' to the melon. Try and keep the background as plain as possible to make it easier to see the outline once the image has been sent to your calculator.

Step 2:

The calculator screen has a 4:3 aspect ratio, the background image however is mapped to 83:133. To ensure the watermelon doesn't get distorted, use imaging software to crop your watermelon image to the 83:133 ratio. Try and crop as close to the edge (major axis) of your watermelon image. The actual watermelon's dimensions are unlikely to be in the same ratio as your calculator screen, this will most likely leave space at the top and bottom of the image. Don't worry too much about the resolution of your image, the TI software will reduce the image resolution to suit the calculator.

Step 3:

Launch the TI-ConnectCE software and connect your calculator to the computer. Select your handheld in the software and the contents icon immediately below the camera icon. A complete list of the content on your calculator is displayed, including any images currently stored on your calculator.



Locate the picture on your computer and drag it into the contents list. You will be prompted to select the image number, then select SEND.

Step 4:

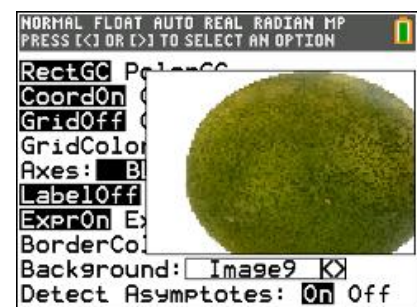
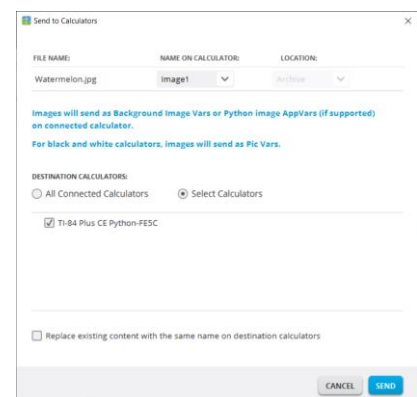
The watermelon image can now be set up as a background for the graph screen. On the calculator press: $\boxed{2\text{nd}}\boxed{\text{format}}$. Navigate down to **Background** and select your image.

Once your image has been selected, press the $\boxed{\text{graph}}$ key to the axis and image.

Note: The axis scale **must** be set up to accurately reflect the physical dimensions of the watermelon. The window settings must also remain 'square', that is to say the scale in both the x and y direction must be the same.

Step 5:

Measure and record the length (major axis) and height (minor axis) of your watermelon.



On the calculator, press: $\boxed{\text{zoom}}$ [6] (standard) then $\boxed{\text{zoom}}$ [5], this will ensure the cartesian plane has the same scale in the x and y direction and that the origin is in the centre of the screen.

Whilst on the graph screen, press the $\boxed{\text{trace}}$ key and navigate to the left most point of the watermelon image, record the x co-ordinate. In the sample shown here the x co-ordinate is: -12.195

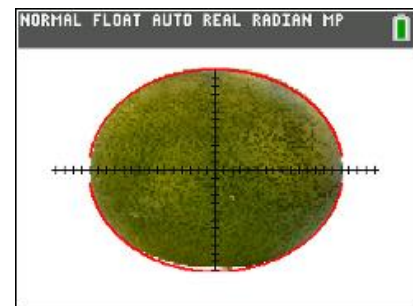
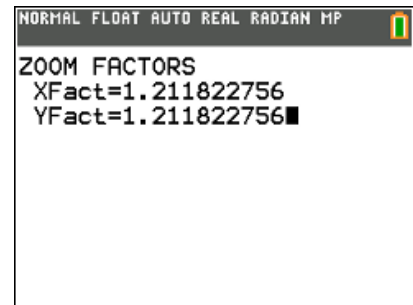
Now navigate to the right most point of the watermelon image, once again record the x co-ordinate.

Divide the measurement for the major axis (cm) by the difference in the two x co-ordinates, the result is the scale factor for your window settings.

The window settings can be changed directly, alternatively the zoom factors can be set to make the adjustments easier. Press $\boxed{\text{zoom}}$ and navigate across and select: **memory**, select option [4] Set Factors. Make sure the XFact and YFact are the same, this ensures zooming in or out will remain 'square'. When the zoom-in or zoom-out options are used the calculator will automatically adjust the x and y scales accordingly.

Return to the graph screen, place the cursor at the origin, then press: $\boxed{\text{zoom}}$ [3] zoom-out and press $\boxed{\text{enter}}$.

To confirm that the scale has been set correctly, use the Trace option again to locate the x coordinates of the extremities of the watermelon. The difference between these extreme values should be equal to the length of the major axis. Do the same for the extremities of the y coordinates for the watermelon, the difference between these values should be equal to the measurement for the minor axis of the watermelon.



Question: 2.

The general equation for an ellipse is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Transpose the equation to make Y the subject. Determine an equation to model the edge of the watermelon storing the corresponding values in 'a' and 'b'.

Note: As the equation is a relation, use Y_1 to model the top of the watermelon and $Y_2 = -Y_1$ to model the bottom.

You may also need to translate your function, alternatively, move the origin to the 'centre' of the watermelon by adding/subtracting values to the Xmin, Xmax, Ymin, Ymax values.

Question: 3.

Your teacher will cut the watermelon into slices such that each slice resembles 'short' cylinder, a cross-section of the watermelon. Use a ruler to measure the dimensions of your cylinder (watermelon slice) and determine the corresponding volume.

Question: 4.

Collect the volume of watermelon each slice from each of your classmates and hence determine an approximate volume for the original watermelon.



Question: 5.

Determine the location of **your** slice of watermelon on the graph. Explain how the radius of your slice of watermelon relates to the function.

Question: 6.

Explain what this expression is doing:

$$\pi \sum_{-a}^a y^2,$$

where 'a' represents the nearest integer value for the x-axis intercepts of the function modelling the watermelon.

Try this on your calculator (see opposite)

Question: 7.

Explain how this expression (below) relates to the expression in the previous question.

$$\pi \int_{-a}^a y^2 dx$$

NORMAL FLOAT AUTO REAL RADIAN MP	
Y→B	1.211822756
X→A	12.11822756
$\pi \int_{X=-\text{int}(A)}^{\text{int}(A)} (Y_1^2)$	15.22167608
$\pi \sum_{X=-\text{int}(A)}^{\text{int}(A)} (Y_1^2)$	9363.727144

NORMAL FLOAT AUTO REAL RADIAN MP	
Y→B	12.11822756
X→A	15.22167608
$\pi \int_{-A}^A (Y_1^2) dX$	9363.727144

Application

A measuring jug (shown) contains measurements in 100ml increments, up to 500ml.

- Insert a copy of the image into your calculator.
- Scale your calculator window settings accordingly.
- Model the side(s) of the measuring jug using a linear function.
- Rotate the function around the y – axis to form a solid of revolution, and check the approximated volume for the 500ml marking.
- Determine the distance between the 100ml – 200ml – 300ml – 400ml markings.

