Bewildered Babies

ID: 9975

Time required
45 minutes

Activity Overview

*Students will explore combinations, factorial expressions, and probabilities. They will test the limits of the combinations formula by applying it a labeling situation. In Problem 1, after making charts and using logic to list possible label arrangements, students compare their results with the output of the combinations formula and **nCr** command. They may formalize the results of this comparison by writing a piecewise function for the number of label arrangements in an optional extension. In Problem 2, they write and simplify factorial expressions to calculate probabilities involving combinations.*

Topic: Data Analysis and Probability

- *Use the Fundamental Counting Principle to calculate the number of outcomes in a sample space.*
- *Use factorial notation to express the number of permutations and combinations of n elements taken r at a time.*
- *Evaluate expressions involving factorials to compute the number of outcomes in a sample space.*
- *Use factorial notation to express the theoretical probability of a simple event in a finite sample space.*
- *Evaluate expressions involving factorials to compute the theoretical probability of a simple event.*

Teacher Preparation

- *This activity is appropriate for an Algebra 1 classroom. Students should have experience with the Fundamental Counting Principle as well as calculating the probability of complex events.*
- *This activity is intended to be student-centered. Students may work individually or in pairs.*
- ***To download the BABYLIST program and student worksheet, go to education.ti.com/exchange and enter "9975" in the quick search box.***

Associated Materials

- *Alg1Week28_BewilderedBabies_Worksheet_TI84.doc*
- *BABYLIST.8xp*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Too Many Choices (TI-84 Plus) — 11762*
- *Probability (TI-84 Plus and TI-Navigator) — 2146*
- *Permutations and Combinations (TI-84 Plus and TI-Navigator) — 1756*
- *What's Your Combination? (TI-84 Plus) — 10126*

Problem 1 – Finding combinations

The situation: Four babies born during the same night in the same hospital were labeled with four identification bracelets. Somehow, the bracelets were mixed up, and only two are correct. How many different ways can this happen?

Students will make a chart to help answer this question. Imagine the babies names are A, B, C, and D. In the chart to the right, all of the different ways to label the babies so that 2 bracelets are correct and 2 are incorrect are listed.

A	B	C	D
A	*B*	*D*	*C*
A	*C*	*B*	*D*
A	*D*	*C*	*B*
B	*A*	*C*	*D*
C	*B*	*A*	*D*
D	*B*	*C*	*A*

Next, students will look at the situation with 5 babies where 3 were labeled correctly and 2 were labeled incorrectly. The chart to the right shows all of the possibilities for this situation.

A	B	C	D	E
A	*B*	*C*	*E*	*D*
A	*B*	*E*	*D*	*C*
A	*B*	*D*	*C*	*E*
A	*E*	*C*	*D*	*B*
A	*D*	*C*	*B*	*E*
A	*C*	*B*	*D*	*E*
E	*B*	*C*	*D*	*A*
D	*B*	*C*	*A*	*E*
C	*B*	*A*	*D*	*E*
B	*A*	*C*	*D*	*E*

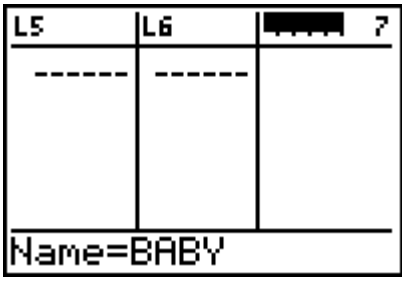
Students should find that while creating a chart is useful in displaying all of the possibilities, it can quickly become a cumbersome task! They will learn the formula for finding the number of combinations later in this activity.

Students should run the program **BABYLIST**. The program creates three lists:

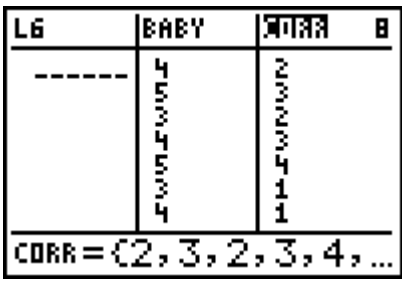
- **BABY**, which contains the total number of babies
- **CORR**, which contains that number of babies labeled correctly, and
- **WAYS**, which shows the number of different ways to arrange the labels in each situation

To view the lists, students should press [STAT] and select **1:Edit…** to enter the **List Editor**. Arrow up to the title row, then over to the right until you reach a list with no title.

Type **BABY** (the calculator is already in ALPHA mode) and press [ENTER]. This recalls the list of the total numbers of babies to the **List Editor**.

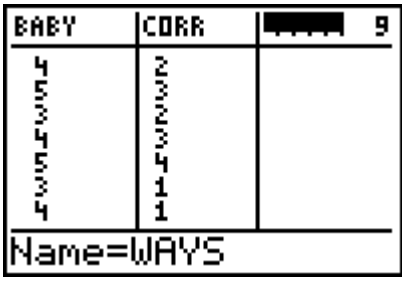


Arrow over to the next column, type **CORR** and press ENTER to recall the list of the numbers of babies labeled correctly to the **List Editor**.



Arrow over to the next column, type **WAYS** and press ENTER to recall the list of the numbers of different ways to arrange the labels in each situation. Only the first three entries are given. Students should either create charts or use logic to complete the table.

After students have filled in the table, introduce the term combinations.

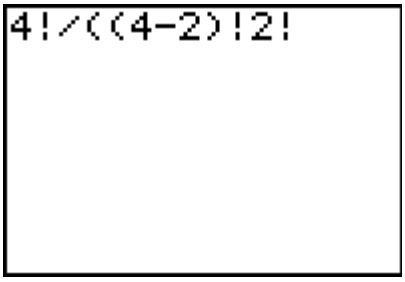


To understand combinations better, start with a simpler problem.

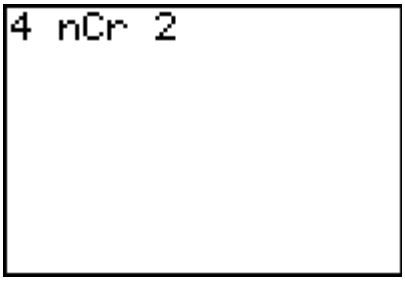
A group of 4 students chooses 2 members to represent the group in a presentation. How many different ways can the group choose?

Students should use the combinations formula to find the numbers of combinations of 2 students chosen from a group of 4.

To calculate a factorial with the calculator, press MATH, arrow over to the **PRB** (Probability) menu, and choose the **!** operator.

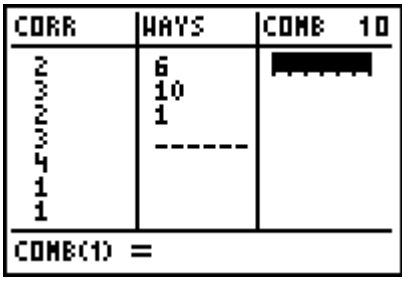


You can also use the **nCr** command (found in the **Math > PRB** menu) to find the number of combinations.

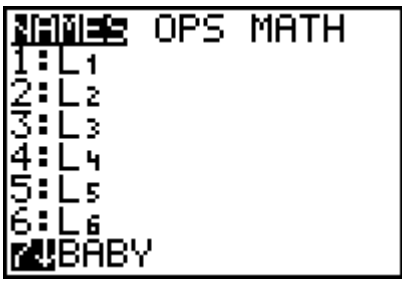


Combinations explain the number of different ways to label the babies? The **nCr** command can be used to find the number of different ways to choose which babies to label correctly from the total number of babies. Press STAT select **1:Edit…** to open the **List Editor** again.

Arrow over to the next list, type **COMB** and press ENTER to title the list.



While the title of the list is highlighted, press 2nd STAT to open the **List > Names** menu. Choose the list **BABY** from the list of names.

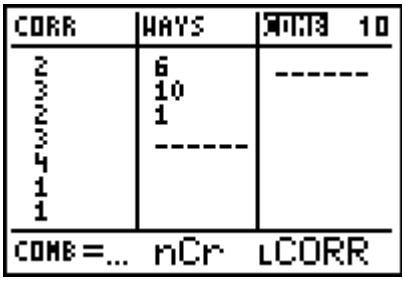


Choose the **nCr** command from the **Math > PRB** menu.

Press 2nd STAT to open the **List > Names** menu. Choose the list **CORR** from the list of names.

Press ENTER to calculate **BABY nCr CORR** and store the results in the **COMB** list.

Students should find that when fewer than 2 babies are labeled incorrectly, the number of ways is not equal to the number of combinations.

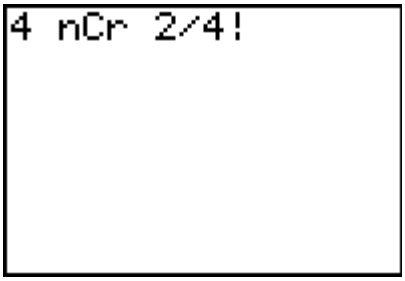


Student Solutions

1. There are 6 ways to label the babies so that 2 bracelets are correct and 2 are incorrect.
2. There are 10 ways to label 5 babies so that 3 are labeled correctly and 2 are labeled incorrectly.
3. No. If 2 of the 3 babies are labeled correctly, the third must be also.
4. There is one way to label 3 babies so that 2 of them are labeled correctly.
5. List **WAYS** should read {6, 10, 1, 1, 1, 3, 4, 5, 1, 1}.
6. The students can choose 2 students from the group of 4 in 6 different ways.
7. When fewer than 2 babies are labeled incorrectly, the number of ways is not equal to the number of combinations.
   Because there is only one correct label for each baby, it is not possible to label all but one of the babies correctly.

Problem 2 – Finding probabilities

Review the definition of probability with students. Combinations can also be used to find probabilities. For example, if the bracelets of 4 babies were mixed randomly, what is the probability that two will be correct and two will be incorrect?



Students should use the calculator's **factorial** and **nCr** commands to check their answers.

Student Solutions

8. The number of possible outcomes is 4! = 24.

9. The number of favorable outcomes is ${}_4C_2 = 6$.

10. P(2 correct and 2 incorrect) = $\frac{1}{4}$

11. a. P(all correct) = $\frac{1}{4!} = \frac{1}{24}$

b. P(3 correct) = $\frac{1}{4!} = \frac{1}{24}$

c. P(exactly 1 correct) = $\frac{{}_4C_1}{4!} = \frac{\frac{4!}{(4-1)!1!}}{4!} = \frac{1}{3!} = \frac{1}{6}$

d. P(all incorrect) = 1 – P(exactly 1 correct) – P(exactly 2 correct) – P(all correct)
= $1 - \frac{1}{6} - \frac{1}{4} - \frac{1}{24} = \frac{24}{24} - \frac{4}{24} - \frac{6}{24} - \frac{1}{24} = \frac{13}{24}$

e. P(at least 1 incorrect) = 1 – P(all correct) = $1 - \frac{1}{24} = \frac{23}{24}$

f. P(at least 1 correct) = 1 – P(all incorrect) = $1 - \frac{13}{24} = \frac{11}{24}$

Extension – Writing a piecewise function

Student Solutions

$$f(x) = \begin{cases} 13 & \text{if } x = 0 \\ \frac{4!}{(4-x)!\,x!} & \text{if } x = 1, 2 \\ 1 & \text{if x=3, 4} \end{cases} \qquad g(x) = \begin{cases} 94 & \text{if } x = 0 \\ \frac{5!}{(5-x)!\,x!} & \text{if } x = 1, 2, 3 \\ 1 & \text{if x=4, 5} \end{cases}$$

Note: The most difficult part of writing this function is calculating the number of ways when $x = 0$. Have students refer back to the table as well as the probabilities they calculated. One way is as follows: if there are 24 possible arrangements, and the probability of none of the bracelets being incorrect is $\frac{13}{24}$, there must be 13 different ways to label the babies so that none of the bracelets are correct.