Great Expectations

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Answers

7 8 9 10 **11** 12

TI-Nspire Investigation Student 120 min

Introduction

Sometimes things just don't live up to their 'expectations'. In this activity you will explore three special dice and determine if they live up to their expectations. These dice may be regular in terms of their shape and form; however each die does not contain the numbers one through to six. The numbers on each die are listed below:

- Red: {3, 3, 3, 3, 3, 6}
- Blue: {2, 2, 2, 5, 5, 5}
- Green: {1, 4, 4, 4, 4, 4}

A simple game is played between two people. Each player selects their own die: Red, Blue or Green. The dice are rolled, the player that rolls the highest number wins.

Question: 1.

Determine the Expected value for each die: Red, Blue and Green.

E(Red) = $3\times\frac{5}{6}+6\times\frac{1}{6}=\frac{7}{2}$ E(Blue) = $2\times\frac{3}{6}+5\times\frac{3}{6}=\frac{7}{2}$ E(Green) = $1\times\frac{1}{6}+4\times\frac{5}{6}=\frac{7}{2}$

Question: 2.

Based on your calculations from Question 1, do you think the game is fair?

If the expected value is representative of the fairness … then the game would be perfectly fair, however as students will soon discover, the expected value is not a suitable measure.

Teacher Notes:

Consider for example a die = {1, 1, 1, 1, 1, 16}. This die would also have an expected return of 3.5 but would lose most games due the quantity of 1's. It doesn't matter how much you win by, so an exceptionally high number on one side of the die will increase the expected value but only provide one option of winning.

Playing the Game

Open the TI-Nspire file: Great Expectations

Navigate to page 1.2 and seed the random number generator using a four digit number of your choosing.

Probability > **Random** > **Seed**

Enter your own, unique four (or more) digit number for the random seed. This ensures your results will be unique.

Select a friend to play against and ask them to choose a dice colour. Once your friend has selected their dice, choose a different colour for yourself.

Author: P. Fox

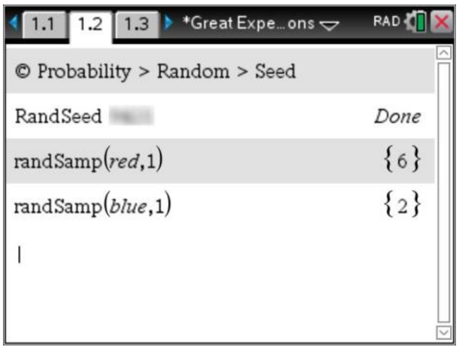
It is time to start playing. A dice roll can be simulated by taking a random sample:

Probability > Random > Sample

Use the VAR key to enter your selected dice colour followed by a comma and a one. The one represents the quantity of samples to be taken.

Compare your randomly generated number with your friends, remember highest number wins!

In the example shown: Red wins.



Question: 3.

Play 10 games and record the results below. Answers will vary.

Game	#1	#2	#3	#4	#5
Colour:					
Colour:					
Winner					
Game	#6	#7	#8	#9	#10
Colour:					
Colour:					
Winner					

Question: 4.

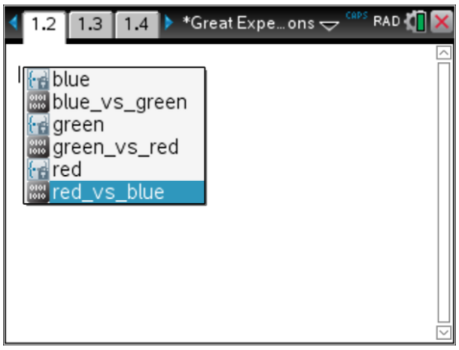
Based on your relatively small sample of 10 games, do you think the game is fair? Discuss.

Ten games does not represent a substantial enough sample to determine if the game(s)/dice are fair…

It is possible to simulate 100's of games all at once. There are three programs:

- Red_vs_Blue
- Blue_vs_Green
- Green_vs_Red

Red_vs_Blue will play a nominated number of games between the red dice and the blue and return the total number of games won by each, similarly for Blue_vs_Green and Green_vs_Red.



Question: 5.

Simulate 100 games of Red vs Blue and record the overall results. From the results decide whether the red or blue dice provides a greater chance of winning.

Answers will vary with regards to the results of the simulation, however the sample is now substantial enough to provide a rough estimate for the probability of a win and to illustrate that the Red die is better than the Blue.

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Question: 6.

Simulate 100 games of Blue vs Green and record the overall results. From the results decide whether the blue or green dice provides a greater chance of winning.

As per Question 5, results will vary from simulation to simulation, however 100 trials usually enough to determine that the Blue die is better than the Green.

Question: 7.

Based on your results to date; is Green more or less likely to beat Red? Discuss.

Thinking non-transitively, it would seem logical that since Red beats Blue and Blue beats Green … R > B > G then it would follow that Red should beat Green. At this point of course students will probably not be aware of the non-transitive property, these dice are a little bit like Rock – Paper – Scissors.

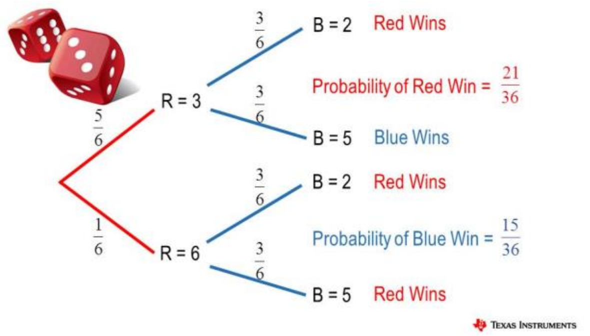
Question: 8.

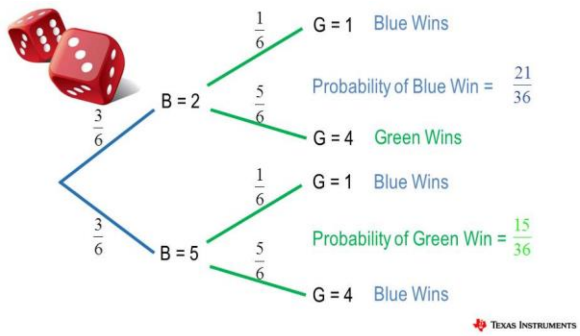
Simulate 100 games of Green vs Red and record the overall results. Discuss the outcome.

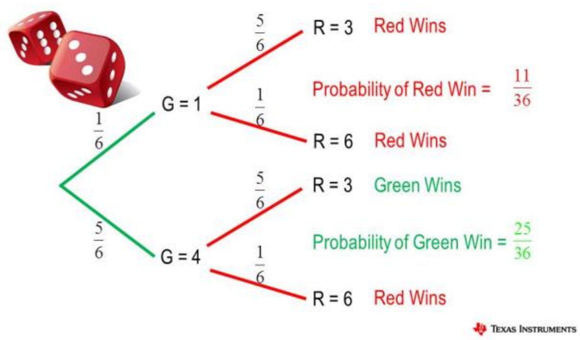
Green wins almost every series of 100 games against Red. So we see that Red beats Blue, Blue beats Green and Green beats Red! (Rock – Paper – Scissors)

Question: 9.

Draw probability tree diagrams for each game: Red vs Blue, Blue vs Green and Green vs Red. Use your tree diagrams to explain the results you obtained for Questions 5, 6 and 8.







Red beats Blue, Blue beats Green and Green beats Red. These results confirm the non-transitive property established using the experimental simulations. Playing this game is therefore unfair for whoever chooses their dice first. If your opponent chooses the Red die, then you should choose Green. If your opponent chooses the Green die, then you should choose the Blue; finally, if your opponent chooses the Blue you should choose Red.

Question: 10.

Explain why the 'expected' value does not help determine which dice is likely to win.

The size of the win is irrelevant; however a single large number on just one side of the die will increase the expected value. The expected value for the three Grime dice has been manipulated in such a way that each one has the same expected value, a lovely illustration that the expected value is not indicative of the likelihood of a win.

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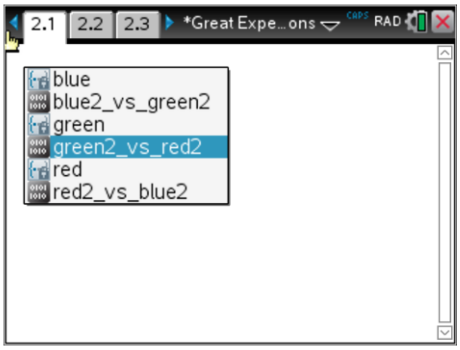
Doubling Up

Navigate to page 2.1

In problem 2 each selected dice is rolled twice and the sum of the results is used to determine the winner. The names of the simulation programs have changed to reflect the changing conditions.

- Red2_vs_Blue2
- Blue2_vs_Green2
- Green2_vs_Red2



Question: 11.

Use the programs to determine approximately probabilities for Blue vs Green, Green vs Red and Red vs Blue games.

Answers will vary, and the accuracy of the results will depend on the number of trials. Using 2000 simulations:

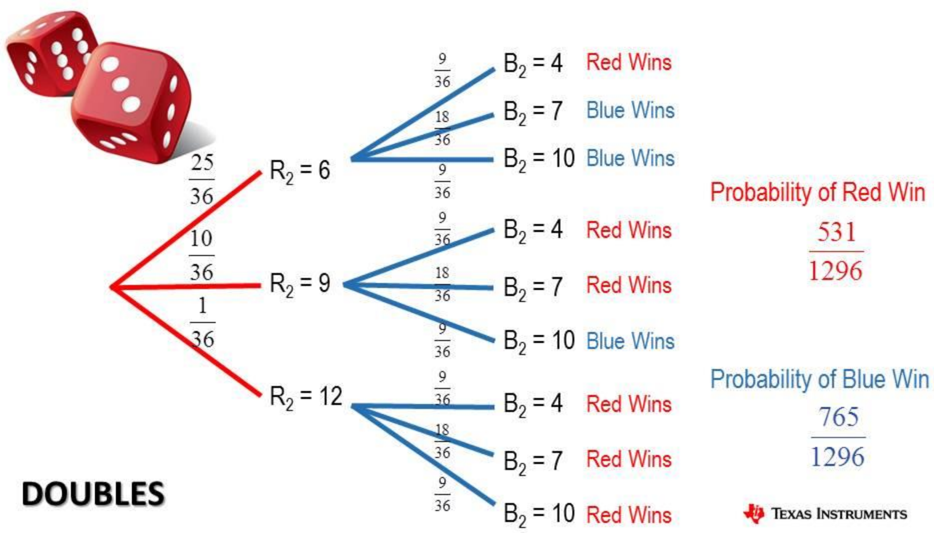
- Red vs Blue … Probability Red Wins ≈ 0.42
- Blue vs Green … Probability Blue Wins ≈ 0.42
- Green vs Red … Probability Green Wins ≈ 0.49

Question: 12.

Use probability to determine the theoretical probabilities for each combination and discuss the results in relation to the original order of the probabilities for single rolls.

The sum totals and respective probabilities of obtaining each sum are listed below:

RED			Blue			Green		
3 +3 = **6**	3 + 6 = **9**	6 + 6 = **12**	2 + 2 = 4	2 + 5 = **7**	5 + 5 = **10**	1 + 1 = **2**	1 + 4 = **5**	4 + 4 = **8**
$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{9}{36}$	$\frac{1}{36}$	$\frac{10}{36}$	$\frac{25}{36}$



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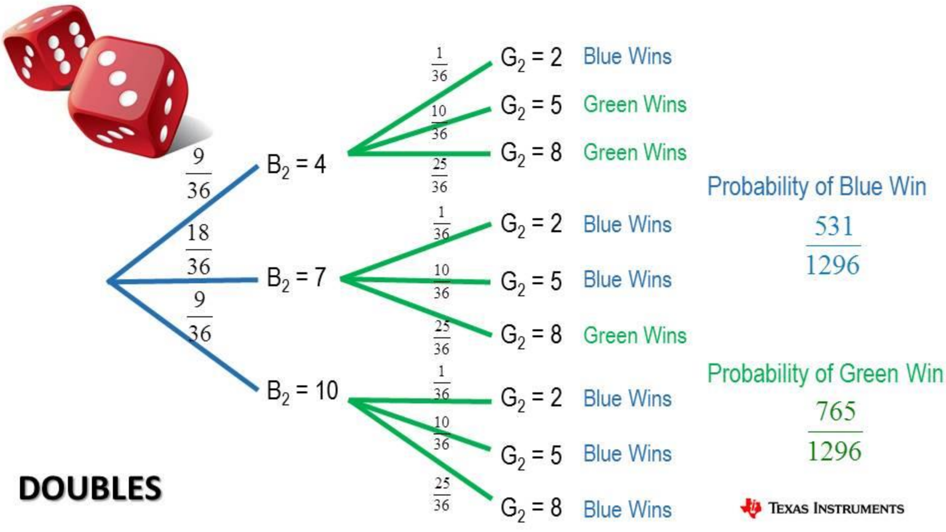

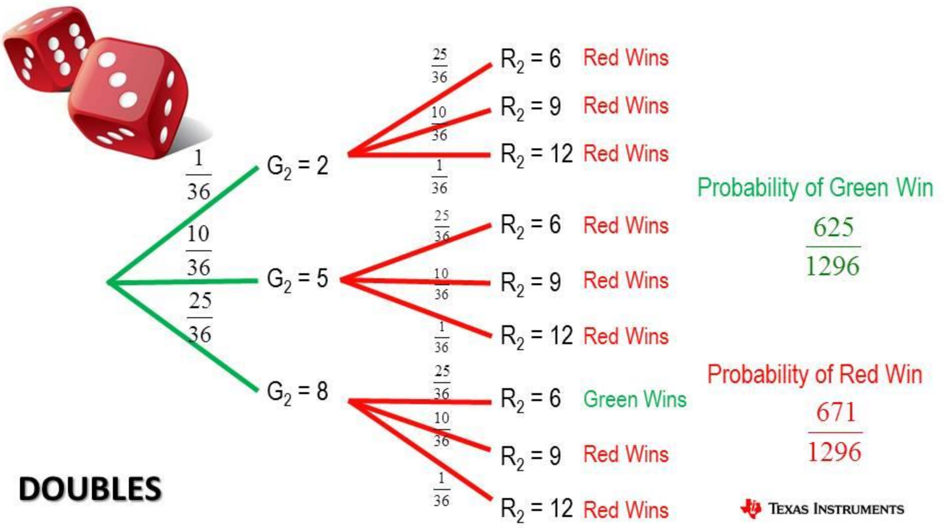

Extension

The dice used in this initial investigation are called Grime dice after a very talented and entertaining mathematician Dr. James Grime. James has a passion for communicating many of the wonders hidden within mathematics, search for him on YouTube or visit his website: http://singingbanana.com/ to find out more about non-transitive dice and other mathematical curiosities.

Similar dice sets have been created by other mathematicians. Efron dice for example are named after the American statistician Brad Efron.

- Blue = {3, 3, 3, 3, 3, 3}
- Red = {0, 0, 4, 4, 4, 4}
- Green = {1, 1, 1, 5, 5, 5}
- Magenta = {2, 2, 2, 2, 6, 6}

Determine your own investigation into this set of non-transitive dice. There are many options to explore here:

- Determine the order, with four dice this means extra possibilities
- Doubles (As per the original investigation)

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- Picking two dice and using the sum
- Picking two dice and using the product
- Consider a three player game and consider scenarios such as: "Your opponents pick the Blue and Green dice, which die should you choose now to give you the best chance of winning?"

No answers are provided here for the investigations option as students can create a myriad of variations.

Teacher Notes

It is relatively easy however to write a calculator program to do an approximate check for most tasks.

Local wins,n:	/variables held in program only
Blue:={2, 2, 2, 5, 5, 5}	/Blue dice defined as list
Green:={1, 4, 4, 4, 4, 4}	/Green dice defined as a list
Request "How many games? ",n	/Input number of games to be trialled
wins:=countIf(randSamp(blue,n)>randSamp(green,n),true)	/Counts how many times blue > green
Disp "Blue won: ",wins	/Displays number of wins for blue
Disp "Green won: ",n-wins	/Displays number of wins for green

If the blue and green dice area already defined, calling the following command on a calculator page:

countIf(randSamp(blue,100)>randSamp(green,100),true)

would generate a random sample of 100 blue dice and green dice and count how many times the blue dice is larger than the green dice.

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