

The Fibonacci Sequence

Student Activity

7 8 9 10 11 12



Introduction

There are an enormous number of amazing sequences and further patterns within each sequence. This investigation looks specifically at the Fibonacci sequence, one of the most famous of all sequences. The sequence is named after Leonardo Pisano (Leonardo of Pisa), his original name was Leonardo Fibonacci.

Fibonacci

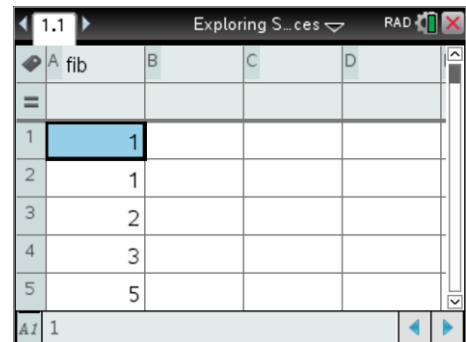
This is one of the most famous sequences: 1, 1, 2, 3, 5, 8, 13 ...

Each number is the sum of the previous two numbers. In its raw form the first two numbers are 1 and 1.

Use either a recursive formula or an appropriate sequence command to generate the first 100 terms of the Fibonacci sequence.

Call the list: FIB

Insert a Calculator Application in preparation for your exploration.



A	B	C	D
fib			
1	1		
2	1		
3	2		
4	3		
5	5		

Question: 1.

Explore the **last** digit of each Fibonacci number from the 61st term onwards. What do you notice?

Note: Any term in the sequence can be recalled by typing: FIB[#] where # represents the term number.

Calculator Tip!



Insert a Notes application and a slider called 'n'. Set the minimum value of the slider to 1, the maximum to 40 with increments of '1'. In two separate and consecutive maths boxes type:

Fib[n],

Fib[n+60]

Question: 2.

What happens when you square consecutive Fibonacci numbers and add them together? Try this for the first couple of Fibonacci numbers and write down the result. Where have you seen this set of numbers before?

Example: $1^2 + 1^2 = 2$

$$1^2 + 2^2 = 5$$

$$2^2 + 3^2 = 13$$

Question: 3.

Square each of the numbers in the Fibonacci sequence. Find the progressive sum of these squared values. Compare this result to the product of consecutive Fibonacci numbers. What do you notice?

Example: $1^2 + 1^2 = 2$

$$1 \times 1 = 1$$

$$1^2 + 1^2 + 2^2 = 6$$

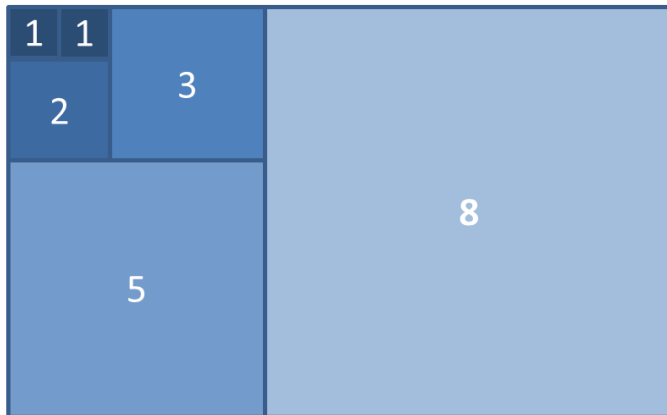
$$1 \times 2 = 2$$

$$1^2 + 1^2 + 2^2 + 3^2 = 15$$

$$2 \times 3 = 6$$

Question: 4.

Study the diagram below. Explain how it is constructed, how it is related to the Fibonacci sequence and how it illustrates the pattern observed in Question 3.

**Question: 5.**


The ratio between consecutive Fibonacci terms is interesting and has its own name.

Explore this ratio as $n \rightarrow \infty$ for $\frac{fib[n+1]}{fib[n]}$.

Calculator Tip!

Insert another Notes application and a slider called 't'. Set the minimum value of the slider to 1, the maximum to 100 with increments of '1'. In a maths boxes type:

$$\text{Approx}(\text{Fib}[t+1] / \text{Fib}[t])$$

Once you believe you know enough about the value store it as ϕ on the calculator. The ϕ symbol is located in the symbols palette (ctrl + ), once stored the value can be retrieved from the VAR menu.

- Square the number ϕ . What do you notice?
- Use the result from part(a) to write an algebraic expression, solve the expression and hence determine the exact value of ϕ .
- Calculate the reciprocal of ϕ , what do you notice?
- Test this quantity: $\phi^1 + \phi^2 = \phi^3$, does it work for any other powers?

The Fibonacci sequence also appears in the 'shallow diagonals' of Pascal's triangle! If the numerical patterns within the Fibonacci sequence have not convinced you about the amazing patterns and beauty, try searching for images related to the Fibonacci sequence on the World Wide Web. Nature is abundant with examples of this amazing sequence.