Parallel & Perpendicular Lines

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Teacher Notes Answers

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TI-Nspire™ Investigation Student 50 min

Introduction

Parallel and perpendicular lines in geometry are everywhere, but how can you make two equations parallel or perpendicular?

Scan the QR code or use the link to watch a video to explore parallel and perpendicular lines.

http://bit.ly/paraperp

Question: 1.

Determine the equation to the straight line that is parallel to $y=2x-1$ passing through the point (4, 1).

Answer: $y=2(x-4)+1$ which simplifies to: $y=2x-7$

Question: 2.

Determine the equation to the straight line that is parallel to $y=-x+2$ passing through the point (1, 3).

Answer: $y=-(x-1)+3$ which simplifies to: $y=-x+4$ or $y=4-x$

Question: 3.

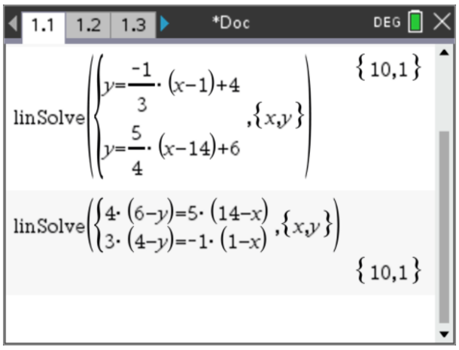
A trapezium ABCD has vertices: A(-3, 2); B(4, 5); C(8, 3) and D(-6, -3). Identify the pairs of parallel sides.

Answer: AB is parallel to CD: $M_{AB}=\dfrac{5-2}{4-{}^{-}3}=\dfrac{3}{7}$ and $M_{BC}=\dfrac{3-{}^{-}3}{8-{}^{-}6}=\dfrac{6}{14}=\dfrac{3}{7}$

Question: 4.

A parallelogram has vertices: A(1, 4); B(5, 9); C(14, 6) and D(d_x, dy). Determine the coordinates of point D.

Answer: $M_{AB}=\dfrac{9-4}{5-1}=\dfrac{5}{4}$ and $M_{BC}=\dfrac{9-6}{5-14}=-\dfrac{1}{3}$



Students could determine the equations for AD and CD and solve their point of intersection using simultaneous equations. See solution shown opposite:

Alternatively, students can set up linear equations for the gradient expressions with d_x, d_y

$$M_{CD}=\frac{6-d_y}{14-d_x}=\frac{5}{4} \text{ and } M_{CD}=\frac{4-d_y}{1-d_x}=-\frac{1}{3}$$

Solution: d_x = 10 and $d_y=1$

Question: 5.

Determine the equation to the straight line that is perpendicular to $y=2x-1$ passing through the point (4, 1)

Answer: $y=-\tfrac{1}{2}(x-4)+1$ which simplifies to: $y=-\tfrac{1}{2}x+3$

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Question: 6.

Points A, B & C have coordinates: (2, 5), (13, 3) and (p, 9) respectively. Line AC is perpendicular to BC.

a) Determine the value(s) for p.

Answer: $M_{AC} \times M_{AB} = -1$ $\therefore$ $p = 5$ or 10

$$\frac{9-5}{p-2} \times \frac{9-3}{p-13} = -1$$

$$-24 = p^2 - 15p + 26$$

$$0 = (p-10)(p-5)$$

b) Find the coordinates of the midpoint of A and B. Label this as point D.

Answer: Midpoint: $\left(\frac{2+13}{2}, \frac{5+3}{2}\right) = \left(\frac{15}{2}, 4\right)$

c) Show that distances: AD, BD and CD are all equal.

Answer: $d_{AD} = \sqrt{(2-7.5)^2 + (5-4)^2} = \frac{5\sqrt{5}}{2} \approx 5.59$

$$d_{BD} = \sqrt{(13-7.5)^2 + (3-4)^2} = \frac{5\sqrt{5}}{2} \approx 5.59$$

$$d_{CD} = \sqrt{(10-7.5)^2 + (9-4)^2} = \frac{5\sqrt{5}}{2} \approx 5.59$$ AND

$$d_{CD} = \sqrt{(5-7.5)^2 + (9-4)^2} = \frac{5\sqrt{5}}{2} \approx 5.59$$

Question: 7.

Points A(1, 1), B(14, 3), C(10, 10) and D(2, 9) form a quadrilateral.

a) Let P, Q, R and S be the midpoints of AB, BC, CD and DA respectively, determine the coordinates of point P, Q, R and S.

Answer: P(7.5, 2); Q(12, 6.5); R(6, 9.5) and S(1.5, 5)

b) PQRS forms a quadrilateral, show that this quadrilateral is a parallelogram.

Answers: $M_{PQ} = \frac{6.5-2}{12-7.5} = 1$; $M_{QR} = \frac{9.5-6.5}{6-12} = -\frac{1}{2}$;

$M_{RS} = \frac{9.5-5}{6-1.5} = 1$; $M_{SP} = \frac{2-5}{7.5-1.5} = -\frac{1}{2}$.

As $M_{PQ} = M_{RS}$ and $M_{QR} = M_{SP}$ the quadrilateral is a parallelogram.

c) Create your own set of points A, B, C and D such that they form a quadrilateral. Determine the coordinates for P, Q, R and S for your quadrilateral. Show that PQRS is also a parallelogram.

Answer: Answers will vary, forming a quadrilateral from the midpoints of a quadrilateral will always generate a parallelogram. This can be proved easily using coordinate geometry.

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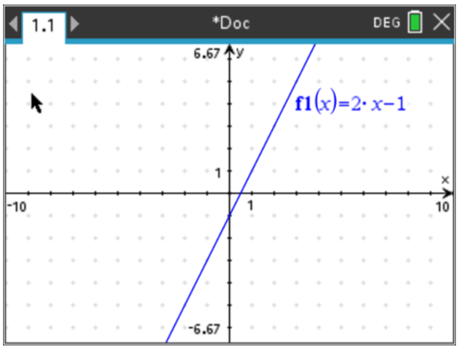
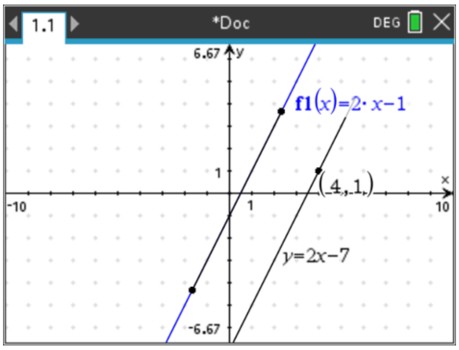
Teacher Notes:

Students should be encouraged to use their calculator to check answers to selected questions and see which approach is more efficient.

Example: Question 1:





Students can graph the line y = 2x – 1, however you cannot use the Geometry tools to construct a parallel line, to an equation.

However, students can use the Point On tool to place two points on the graph.

Use the segment tool to draw a line connecting the two points.

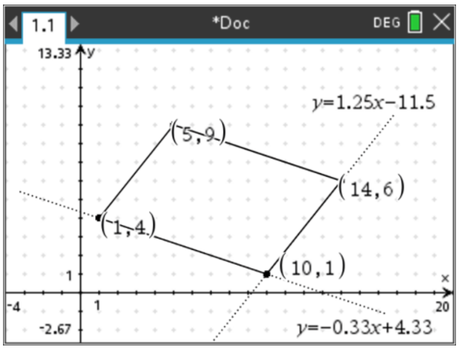
Use the Point (Coordinates) and plot a point at (4, 1)

Now students can use the Geometry tools to construct a parallel line (to the graph of y = 2x – 1, courtesy of the function) passing through the point (4, 1).

The equation can be display to get the result: y = 2x – 7.

Students should note that whilst they *can* get an answer by relying completely on the calculator, in this situation it is much slower, and not necessary.

Example: Question 4:

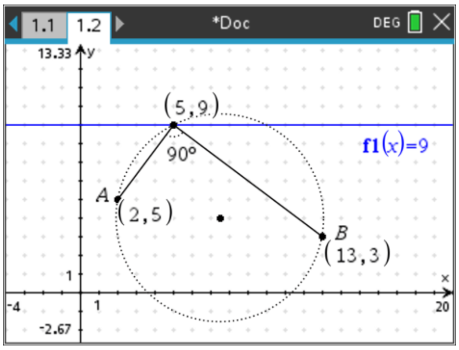


Students can use the calculator to help draw an accurate diagram and also gain a visually accurate representation of the situation.

The solution here was initially created using two line segments with start and end points to match the polygon. Two parallel lines were then added that passed through the relevant vertices.

The point of intersection (10, 1) was then obtained. Whilst this takes almost the same time, it has the added bonus of providing an accurate visual representation.

Example: Question 6:



This question ties in nicely with the Circumcentre activity. The points A and B can be generated using the keyboard shortcut P.

A line corresponding to y = 9 is included as point C contains the ordinate 9. Measure the angle on the two segments can provide a good approximation for the solution(s).

However, once the midpoint is included we see the familiar circle geometry that also ties in with the circumcentre of a triangle where the centre of the circle is on one of the sides of the triangle!

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