Reflections

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teachers Notes and Answers

7 8 9 **10** **11** 12

TI-Nspire™ | Investigation | Student | 50 min

Introduction

Have you ever noticed how the word "Ambulance" is written on the front of the vehicle? Why is it written this way?

In this activity you will reflect points and graphs on the Cartesian plane. Three different reflections will be considered:

- y axis
- x axis
- $y = x$

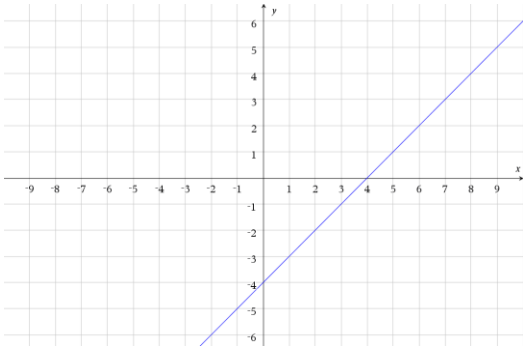
Watch the video using the QR code or short-link to help explore and answer the questions in this activity.

https://bit.ly/reflecting-points

Teacher Notes: Encourage students to use their calculator to check their answer(s) by using the corresponding reflection techniques demonstrated in the video. If point A is on the function then A′ should be on the reflected function.

Question: 1.

The line: $y = x - 4$ is shown opposite.



a) Write down the coordinates of the x and y axes intercepts.
Answers: (4, 0) & (0, -4)

b) The line is reflected in the y axis. Write down each of the following:

i. y axis intercept.
Answer: (0, -4) It's on the mirror so the point is invariant.

ii. x axis intercept.
Answer: (-4, 0)

iii. The equation to the reflected line.
Answer: $y = -x - 4$.

Teacher Notes:
For b(iii), students may use their answers to parts (i) and (ii) to determine the equation, however the idea is for them to simply precede x with a negative sign. As questions get more challenging, students will be much better off with their reflection (transformation) knowledge.

c) The line is reflected in the x axis. Write down each of the following:

i. y axis intercept.
Answer: (0, 4)

ii. x axis intercept.
Answer: (-4, 0) It's on the mirror so the point is invariant.

iii. The equation to the reflected line.
Answer: $y = -x + 4$ or $y = 4 - x$ or $-y = x - 4$

Author: P. Fox

d) The line is reflected in the line: $y = x$. Write down each of the following:

i. y axis intercept.
Answer: (0, 4)

ii. x axis intercept.
Answer: (-4, 0)

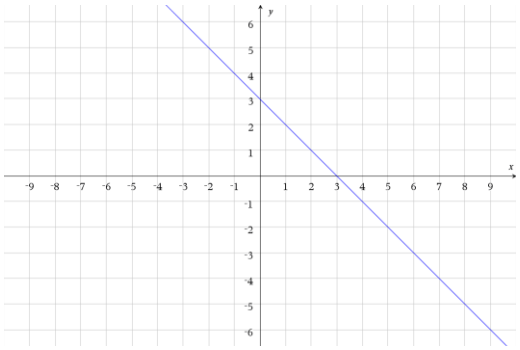
iii. The equation to the reflected line.
Answer: $y = x + 4$ or $x = y - 4$

Teacher Notes:

Students using 'swap x and y' may find d(iii) easier to do first, however students that visualise or use the calculator to do the reflection should be able to answer d(i) and d(ii) easily.

Question: 2.

The line: $y = 3 - x$ is shown opposite.



a) Write down the coordinates of the x and y axes intercepts.
Answers: (3, 0) & (0, 3)

b) The line is reflected in the y axis. Write down each of the following:

i. y axis intercept.
Answer: (0, 3) It's on the mirror so the point is invariant.

ii. x axis intercept.
Answer: (-3, 0)

iii. The equation to the reflected line.
Answer: $y = x + 3$.

c) The line is reflected in the x axis. Write down each of the following:

i. y axis intercept.
Answer: (0, -3)

ii. x axis intercept.
Answer: (3, 0) It's on the mirror so the point is invariant.

iii. The equation to the reflected line.
Answer: $y = x - 3$

d) The line is reflected in the line: $y = x$. Write down each of the following:

i. y axis intercept.
Answer: (0, 3)

ii. x axis intercept.
Answer: (3, 0)

iii. The equation to the reflected line.
Answer: $y = 3 - x$

Teacher Notes: Sometimes the reflection looks the same as the original. The line is perpendicular to the 'mirror'. Students could be asked to nominate other functions that would look the same if reflected on the line $y = x$ or lines that would appear the same if reflected in the x or y axis.

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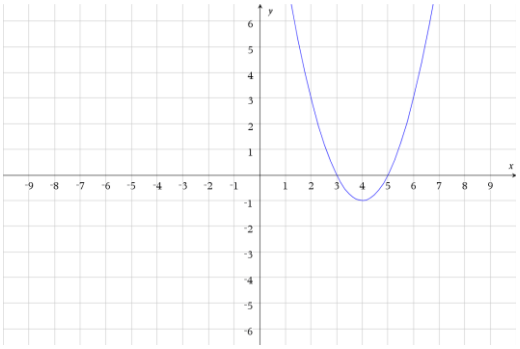
Question: 3.

The graph of $y = x^2 - 8x + 15$ is shown opposite.



a) Write down the coordinates of the x and y axes intercepts.
Answers: (3, 0) & (5, 0) & (0, 15)

b) The graph is reflected in the y axis. Write down each of the following:

i. y axis intercept(s).
Answer: (0, 15)

ii. x axis intercept(s).
Answer: (-3, 0) & (-5, 0)

iii. The equation to the reflected graph.
Answer: $y = x^2 + 8x + 15$. Encourage students to use: $y = (-x)^2 - 8(-x) + 15$.

c) The graph is reflected in the x axis. Write down each of the following:

i. y axis intercept(s).
Answer: (0, -15)

ii. x axis intercept(s).
Answer: (3, 0) & (5, 0)

iii. The equation to the reflected graph.
Answer: $-y = x^2 - 8x + 15$ or $y = -x^2 + 8x - 15$

d) The graph is reflected in the line: $y = x$. Write down each of the following:

i. y axis intercept(s).
Answer: (0, 3) & (0, 5)

ii. x axis intercept.
Answer: (15, 0)

iii. The equation to the reflected graph.
Answer: $x = y^2 - 8y + 15$ - Students should use the relational graphing tool.

Teacher Notes: Encourage more able students to re-write the equation as $y = 4 \pm \sqrt{x+1}$.

Extension

Question: 4.

A linear graph is invariant when reflected in the x axis. Determine a possible equation for such a linear function.

Answer: The graph would be of the form $x = a$. (Vertical line)

Question: 5.

A linear graph is invariant when reflected in the y axis. Determine a possible equation for such a linear function.

Answer: The graph would be of the form $y = a$. (Horizontal line)

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Question: 6.

The graph of $y = \frac{1}{x}$ is invariant when reflected. Determine the equation of the mirror creating the reflection.

Answer: The mirror would be the line $y = x$.

Question: 7.

For what values of *h*, *k* and *r* would the graph of $(x-h)^2 + (y-k)^2 = r^2$ be invariant when reflected in the line $y = x$?

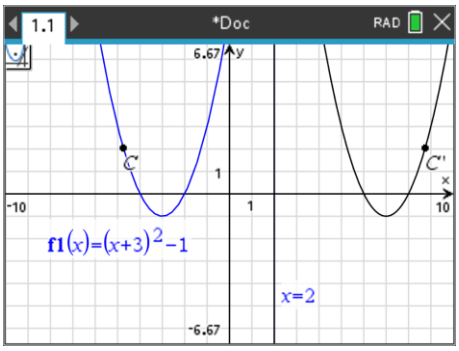
Answer: Reflections in the line $y = x$ result in 'swapping' x and y, so provided $h = k$ the graph will be invariant. The graph will be a circle centred on the line $y = x$.

Question: 8.

The graph of $y = (x+3)^2 - 1$ is reflected in the line $x = 2$, determine the equation for the reflected function.

Answer: Students can use the calculator to explore and 'guess and check' their answer or they might use algebra and their knowledge of reflections and translations. $y = (x-7)^2 - 1$

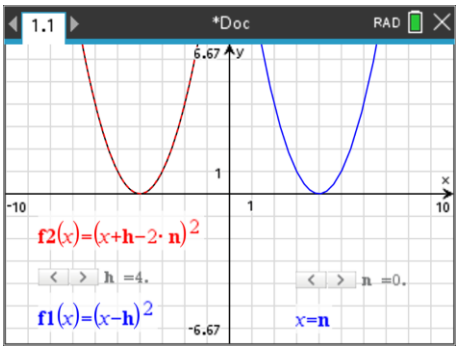
Teacher Notes: Students can graph the original function and the line x = 2. Use the Geometry tool to draw a line on the graph of x = 2. (Required to function as a mirror). If the graph x = 2 is moved the line will automatically move. Use the transformation tool to create a point on the original function and reflect it. Now use the construction tool to create a locus of the original point and its reflection to see the resulting graph. This doesn't produce the 'answer' (equation), but it provides a powerful visual for students to use and interact with to explore the general solution.



Question: 9.

The graph of $y = (x-h)^2$ is reflected in the line $x = n$, determine the equation for the reflected function.

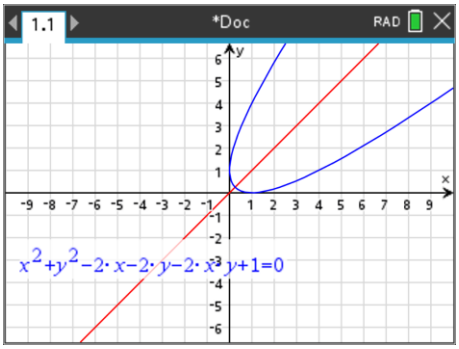
Answer: $y = (x+h-2n)^2$ Students can check their answer through exploration.



Question: 10.

Explain why the graph of: $x^2 + y^2 - 2x - 2y - 2xy + 1 = 0$ would be unchanged by a reflection in the line $y = x$.

Answer: Students may choose to use the relational graphing tool to graph this parabola (symmetrical about the line y = x), however students should note the symmetry of the equation. The equation for a function reflected in the y = x can be obtained by swapping 'x' and 'y' in the equation, doing so in this equation will not result in any changes to the algebraic representation. This was also observed in questions 6 and 7.



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