Points, Lines, and Planes

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will identify parallel lines and planes.
- Students will identify perpendicular lines and planes.
- Students will recognize skew lines.
- Students will discuss the intersection of planes.
- Students will attend to precision (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- parallel lines
- perpendicular lines
- collinear
- noncollinear
- plane
- skew

About the Lesson

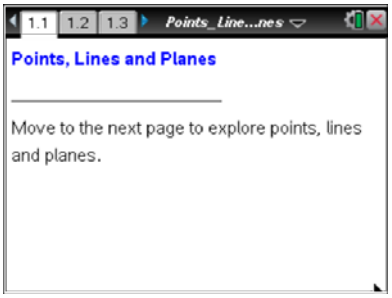
- This lesson involves use of a cube to represent space. Points and segments flash when clicked.
- As a result students will:
  - Click on vertices of a cube to identify planes, including an interior plane.
  - Click on segments of a cube to identify parallel, perpendicular, and skew segments.
  - Rotate the cube to visualize the relationships from different perspectives.
  - Manipulate planes in space to show the intersection of two planes is a line, and the intersection of three planes can be a point or a line.

Prerequisite Knowledge

- Students should be familiar with parallel and perpendicular lines.
- Students should be able to distinguish between collinear and noncollinear points.

TI-Nspire™ Navigator™ System

- Use Teacher Edition computer software or Live Presenter to review student documents and discuss examples as a class.
- Use Quick Poll to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Click to select points and segments

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Points_Lines_and_Planes_Student.pdf
- Points_Lines_and_Planes_Student.doc

TI-Nspire document

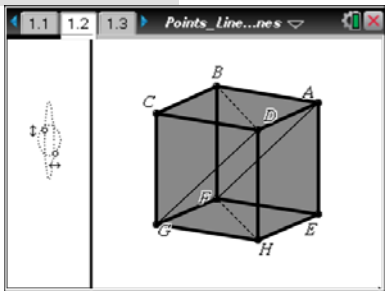
- Points_Lines_and_Planes.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, make sure that they move the cursor until it becomes a hand (☜) getting ready to grab the point. Press ctrl ☜ to grab the point and close the hand (✊).

Move to page 1.2.



The cube is a representation of three dimensions. Although the cube is bounded, planes extend infinitely in all directions.

1. Grab and drag the open circles located on the left side of the screen. What happens when the open circles are moved?

   Answer: The point with the vertical arrow rotates the cube up and down. The point with the horizontal arrow rotates the cube clockwise and counterclockwise.

2. A plane is created by three noncollinear points.
   a. Click on three noncollinear points that are connected to each other by solid segments. Identify the plane formed by these three points. Hover over one of the flashing points. Press tab until it says **polygon** and the name of the plane you chose. Press ☜. The plane will highlight.

      Answers: There are 8 possible answers:

ABCD	*ABFE*
BCGF	*EFHG*
CDHG	*BDFH* (diagonal)
ADHE	*ADFG* (diagonal)

Teacher Tip: Remind students that ctrl tab will move them from one side of the split screen to the other side.

Students need to keep tabbing until the correct name of their polygon shows on the screen. Students will need to press esc at the end of each question to release whatever is selected in preparation for the next exploration.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

b. Identify a plane that is not a face of the cube.

Answer: Any of the diagonal planes are possible answers:

BDFH	*ABGH*	*BCEH*
ACGE	*CDFE*	*ADFG*

3. Press esc to unselect the points from the previous problem. Then, click on the segments of plane *ABCD* to make them flash. Rotate the cube until plane *ABCD* looks like a floor of a room and $\overline{AE}$ is vertical.

a. Identify the planes that form the ceiling and walls of the room.

Answer: Ceiling: *EFGH;* Walls: *CDHG*, *ADHE*, *ABFE*, *BCGF*

b. Which of the planes do you think are parallel? Explain your thinking.

Sample Answers: *ABCD* ∥ *EFGH; CDGH* ∥ *ABFE; ADHE* ∥ *BCGF;* The planes will never intersect. They have no points in common. They are the same distance apart.

c. Which of the planes do you think are perpendicular? Explain your thinking.

Sample Answers: *ABCD* ⊥ *CDHG; EFGH* ⊥ *CDHG; BCGF* ⊥ *CDHG; ABCD* ⊥ *ABFE; EFGH* ⊥ *ABFE; ADHE* ⊥ *CDHG; ABCD* ⊥ *BCGF; EFGH* ⊥ *BCGF; ADHE* ⊥ *ABFE; ABCD* ⊥ *ADHE; EFGH* ⊥ *ADHE; ABFE* ⊥ *BCGF*
The second plane contains a line that is perpendicular to the first plane.

d. Rotate the cube. Are the planes you identified in 3b still parallel? Are the planes you identified in 3c still parallel? Explain your thinking.

Answer: Yes; rotating the cube only changes the perspective, not its properties.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

4. Press esc to unselect the segments from the previous problem. Then, click on $\overline{FB}$. Click on and identify all of the segments that are parallel to $\overline{FB}$.

   Answer: $\overline{CG}$, $\overline{DH}$, $\overline{AE}$

5. Is it possible to have two lines that do not intersect and are not parallel? Explain your thinking.

   Answer: Yes. They would have to lie on different planes.

   Teacher Tip: For example, segments BD and FH are parallel and coplanar. Segments BD and EG are neither parallel nor coplanar**.**

6. Press esc to unselect the segments from the previous problem. Then, click on $\overline{BF}$ and $\overline{CD}$. $\overline{BF}$ and $\overline{CD}$ are skew lines.
   a. What kind of relationship exists between these lines?

      Answer: $\overline{BF}$ and $\overline{CD}$ do not intersect and are not parallel.

   b. Rotate the cube and identify two other segments that are parts of skew lines.

      Sample Answers: Answers will vary. Some of the possibilities are shown below.

$\overline{FB}$ and $\overline{AD}$	$\overline{GC}$ and $\overline{AD}$	$\overline{GC}$ and $\overline{EH}$
$\overline{FB}$ and $\overline{EH}$	$\overline{GC}$ and $\overline{AB}$	$\overline{DH}$ and $\overline{AB}$
$\overline{FB}$ and $\overline{GH}$	$\overline{GC}$ and $\overline{EH}$	$\overline{BC}$ and $\overline{GH}$

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

Teacher Tip: Remind students that the segments are parts of lines in space. Misconceptions arise when using a finite representation of an infinite plane.

7. Press esc to unselect the segments from the previous problem. Then, click on $\overline{BD}$ and $\overline{HF}$. Identify the relationship between these two segments.

   Answer: $\overline{BD} \parallel \overline{HF}$

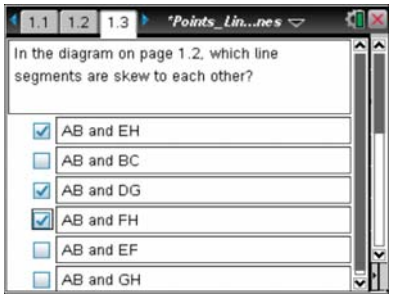
8. Name a segment perpendicular to $\overline{HF}$. Explain your thinking.

   Sample Answers: $\overline{HF} \perp \overline{DH}$; $\overline{HF} \perp \overline{BF}$

   The segments connect at a right angle. The segments are contained on a face of a cube and connect at a vertex.

Move to page 1.3.

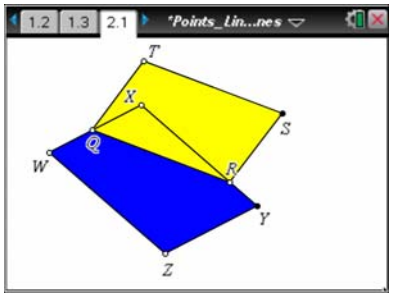
9. Carefully read the question. Select all correct answers to the question. To check your answers, select each question and press **Menu > Check Answer.**



Teacher Tip: Review page 1.3 if necessary. Remind students that skew lines do not intersect and are not coplanar.

Teacher Tip: Page 2.1 shows the intersection of two planes is a line. Grab the open circles and move the planes to show that the intersection remains a line. To help students understand the intersection of planes, guide students through pages 2.1, 2.2, and 2.3. Encourage discussion. Help students interpret different perspectives, so they are able to visualize the planes in three dimensions.

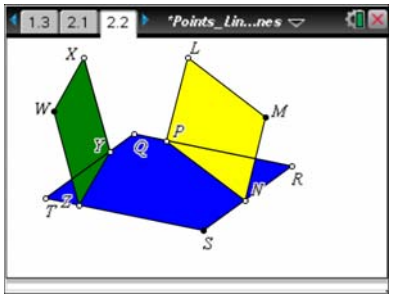
Move to page 2.1.



10. Grab the open circles and move the planes to show the intersection of the planes. What can you say about the intersection of any two planes?

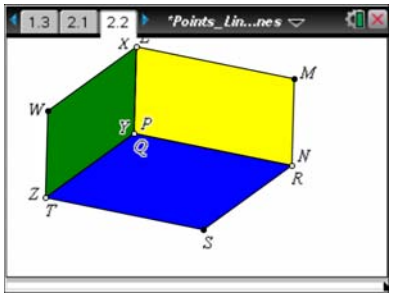
Answer: The intersection of any two planes is a line.

Extension

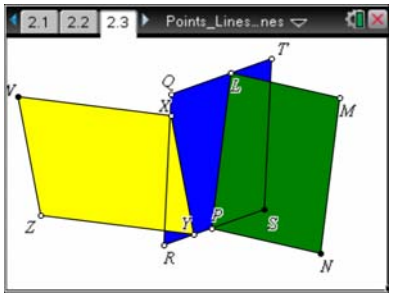


11. Page 2.2 shows that the intersection of three planes can be a point. Grab and move the open points to show that the planes rotate in space, but the intersection of any two planes is a line. The three lines that are formed by the intersection of these planes intersect at one point. Grab and move the open points to show that the intersection of three planes can be a point.

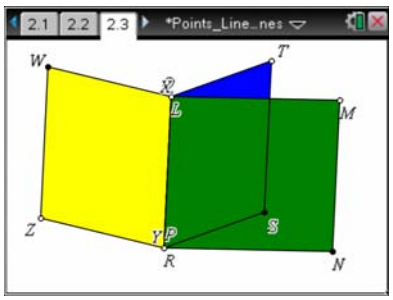
Answer: After moving the points, the graph should look like the one on the right.



12. Page 2.3 shows that the intersection of three planes can also be a line. Grab and move the open points to show that the planes rotate in space, but the intersection of any two planes is a line. Then show that the intersection of three planes can be a line.



Answer: After moving the points, the graph should look like the one on the right.



Wrap Up:

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify parallel lines and planes.
- How to identify perpendicular lines and planes.
- How to recognize skew lines.
- How to describe the intersection of planes.

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Note 1

Questions 1, 2, *Screen Capture*: Use Screen Capture to verify students are able to grab the points to move the cube and find the polygons. Scroll through the screen captures to discuss with the class the eight possible planes.

Note 2

Question 3, *Quick Poll*: Use Quick Poll to gather students' answers to Questions 3a, 3b, and 3c.

Note 3

Question 6, *Screen Capture*: Have students click on segments that are parts of skew lines. Use Screen Capture to share the answers with the class and verify student understanding.