Math Objectives

- Students will recognize the effect of a vertical stretch, vertical compression, and reflection through the *x*-axis on the graph of a function.
- Students will relate the transformation of a graph $y = f(x)$ to the symbolic representation of the transformation. In other words, $y = a \cdot f(x)$ may vertically stretch or compress and/or reflect through the *x*-axis.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practices).

Vocabulary

- vertical stretch
- vertical compression

About the Lesson

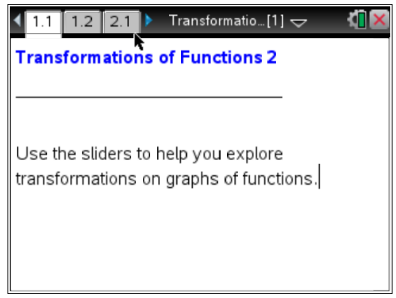
- This lesson involves investigating vertical stretches, vertical compressions, and reflections through the *x*-axis of a function.
- Students will learn to recognize how transformations of the form $y = a \cdot f(x)$ change the graph of $y = f(x)$.

TI-Nspire™ Navigator™

- Use Quick Poll to check student understanding.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Transformations_of_Functions_2_Student.pdf
- Transformations_of_Functions_2_Student.doc

TI-Nspire document

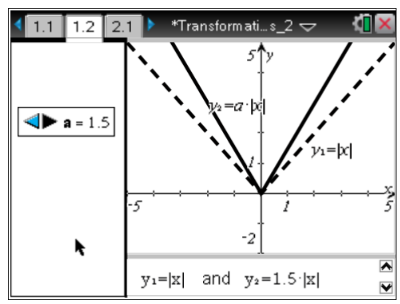
- Transformations_of_Functions_2.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty grabbing and dragging the slider, check to make sure that they have moved the cursor arrow until it becomes a hand (👆) getting ready to grab the slider. They are then to press ctrl [grab key] to grab the slider and close the hand (✊). When they have finished moving the slider, they should press esc to release the slider. Once a function has been graphed, the entry line can be shown by pressing ctrl G.

Tech Tip: Tap the arrows on the slider to change the values.

Move to page 1.2.



1. What happens to the graph of $y_2 = a \cdot f(x)$ as you change the value of **a**?

   <u>**Answer:**</u> The graph of $y_2 = a \cdot f(x)$ stretches or compresses vertically and sometimes opens in the opposite direction.

2. Use the slider to change the value of **a**. Describe how the graph of $y_2 = a \cdot f(x)$ is different from the graph of $y_1 = f(x)$ as the value of **a** changes. Complete the table below.

a	Difference between $y_2 = a \cdot f(x)$ and $y_1 = f(x)$
2	The graph is stretched vertically compared to the graph of $y_1 = f(x)$
2.5	The graph is stretched vertically compared to the graph of $y_1 = f(x)$.
0.5	The graph is compressed vertically compared to the graph of $y_1 = f(x)$.
0.25	The graph is compressed vertically compared to graph of $y_1 = f(x)$.
–1	The graph opens downward and is reflected through the *x*-axis.
–2	The graph is reflected downward and is stretched vertically compared to the graph of $y_1 = f(x)$.
–0.25	The graph is reflected downward and is compressed vertically compared to the graph of $y_1 = f(x)$.
1	The graphs coincide.

3. Based on observations in question 2:
   a. How do you think the graph of $y_2 = a \cdot f(x)$ would compare with $y_1 = f(x)$ for **a** = 5? Explain.

   Answer: The graph of $y_2 = 5 \cdot f(x)$ would open upward and would be stretched vertically compared to the graph of $y_1 = f(x)$.

   b. How do you think the graph of $y_2 = a \times f(x)$ would compare with $y_1 = f(x)$ for **a** = 0.1? Explain.

   Answer: The graph of $y_2 = 0.1 \cdot f(x)$ would open upward and would be compressed vertically compared to the graph of $y_1 = f(x)$.

   c. How do you think the graph of $y_2 = a \times f(x)$ would compare with $y_1 = f(x)$ for **a** = –5? Explain.

   Answer: The graph of $y_2 = -5 \cdot f(x)$ would open downward (reflect through the *x*-axis) and would be vertically compressed compared to the graph of $y_1 = f(x)$.

Teacher Tip: If needed, review the terminology and properties students will encounter in this lesson. For example, you may wish to tell them that:

- A transformation of $y_1 = f(x)$ such as $y_2 = 2f(x)$, where $a > 1$, is called a *vertical stretch*.
- A transformation of $y_1 = f(x)$ such as $y_2 = 0.3f(x)$, where $0 < a < 1$, is called a *vertical compression*.
- A transformation of $y_1 = f(x)$ such as $y_2 = -3f(x)$, where $a < 0$, is called a *reflection through the x-axis*.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

4. Move the slider so that $a = 0$. What happens to the graph of $y_2 = a \cdot f(x)$? Why does this happen?

Answer: The graph lies on the x-axis. This happens because $y_2 = 0$.

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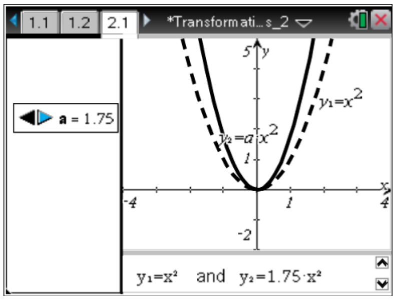

5. Find a value for **a** that will satisfy the given conditions:
   a. The graph of $y_2 = a \cdot f(x)$ is *stretched* vertically compared to the graph of $y_1 = a \cdot f(x)$ and opens in the *same* direction as $y_1 = f(x)$.

   Sample answer: **a** >1. Example, **a** = 2.

   b. The graph of $y_2 = a \cdot f(x)$ is vertically *compressed* compared to the graph of $y_1 = a \cdot f(x)$ and opens in the *opposite* direction from $y_1 = f(x)$.

   Sample answer: $-1 <$ **a** < 0. Example, **a** = –0.25.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

6. a. If the graph of $y_1 = f(x)$ includes the point (1, 3), what corresponding point would be found on the graph of $y_2 = 2 \cdot f(x)$?

   Answer: (1, 6)

   b. If the graph of $y_1 = f(x)$ includes the point (x, y), what corresponding point would be found on the graph of $y_2 = 2 \cdot f(x)$?

   Answer: (x, 2y)

   c. If the graph of $y_1 = f(x)$ includes the point (2, 4), what corresponding point would be found on the graph of $y_2 = -3 \cdot f(x)$?

   Answer: (2, –12)

d. If the graph of $y_1 = f(x)$ includes the point (x, y), what corresponding point would be found on the graph of $y_2 = -3 \cdot f(x)$?

Answer: $(x, -3y)$

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up:

Upon completion of the discussion, ensure that students are able to understand:

- The transformation $y = a \cdot f(x)$ on a function $y = f(x)$ when $a > 1$ will result in a vertical stretch while when $0 < a < 1$ will result in a vertical compression.
- The transformation $y = a \cdot f(x)$ on a function $y = f(x)$ when $a < 0$ will result in a reflection through the x-axis.

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Note 1

Question 3, *Quick Poll:* Use an *Open Response Quick Poll* to have students submit their answers to 3a, 3b, and 3c. Discuss as needed.

Note 2

Question 5, *Live Presenter:* Select a student to be *Live Presenter* and discuss the answer for 5a. Then select another student to be *Live Presenter* and discuss the answer to 5b. If more than one correct answer needs to be illustrated, please do so.

Note 3

Question 6, *Quick Poll:* Use an *Open Response Quick Poll* to have students submit their answers to 6a, 6b, 6c and 6d. Discuss as needed.