ID: 10253

Activity Overview

This activity approaches probability distributions as functions taking information about a dataset as inputs and outputting a set of probabilities. This is analogous to the familiar process of choosing a function to model a set of *x-y* pairs. Students use simulations and graphs to explore was happens when the number of trials of a binomial experiments becomes large number. The normal distribution is introduced, compared with the binomial distribution, and applied.

Topic: Sampling

- Normal Distribution
- Binomial experiments with a large number of trials
- Matching probability distributions to a situation

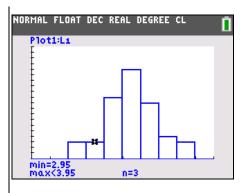
Teacher Preparation and Notes

- It is preferable that students have experience with the binomial probability distribution before beginning this activity. However, this is not essential, as the students can calculate both the binomial and normal distributions with the handheld. Depending on the goals of your class, you may wish to present the algebra behind these calculations in more depth or not at all.
- Before beginning the activity, students should clear all lists and turn off all equations from the Y= screen. To clear the lists, press
 [2nd] [MEM], select ClearAllLists, and then press ENTER on the Home screen.
- To download the program files and student worksheet, go to <u>education.ti.com/exchange</u> and enter "10253" in the keyword search box.

Suggested Related Activities

To download any activity listed, go to <u>education.ti.com/exchange</u> and enter the number in the keyword search box.

- What's Normal, Anyway? (TI-84 Plus family) 9415
- An Approximation to the Binomial Distribution (TI-Nspire technology) — 9296
- PLUMS and the Normal Distribution (TI-84 Plus family with TI-Navigator) — 6082
- Assessing Normalcy (TI-84 Plus family) 11142



This activity utilizes MathPrint[™] functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- ModelingProbabilities_Student.p
 df
- ModelingProbabilities_Student.d
 oc
- COINFLIP.8xp
- DICEROLL.8xp

Click <u>HERE</u> for Graphing Calculator Tutorials.

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Before beginning the activity, explain to students what the two programs do when executed. **COINFLIP** simulates flipping a coin many times and counting the number of heads. **DICEROLL** simulates rolling a die many times and counting the number of dice that land on 2.

Problem 1 – Binomial Experiments with Lots of Trials

Experiment 1

Students will begin by simulating a binomial experiment—tossing a coin 20 times and counting the number of heads—using the program **COINFLIP**. They need to press <u>PRGM</u> to open the Program menu and choose it from the list.

When prompted, students input the number of flips, 20, and the number of trials, 1. After comparing with their classmates, they need to run the program again by pressing ENTER. This time, simulate repeating the experiment (flipping a coin 20 times) 5 times.

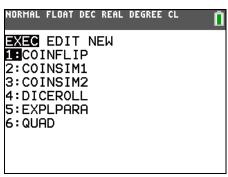
Ask student, *Do the results make sense?* They should expect numbers close to 10 because that's half of 20.

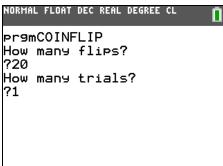
The **COINFLIP** program stores the results of each simulation in list **L1**. To view the results there, students need to press [STAT] [ENTER] to open the List Editor.

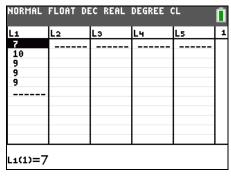
(Note: At this time, advise students clear any data from L2, L3, and L4 if they have not already done so.)

Now students will graph the data in **L1** it as a histogram. Before doing so, students need to clear out any functions from the Y=1 screen.

To set up the histogram, they should press 2nd [STAT PLOT] then ENTER to open **Plot1**. Then adjust the settings of **Plot1** as shown on the worksheet and at the right.









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Each time students need to view the histogram, they should press (and select **ZoomStat** so that it is in

Discussion Questions

an appropriately sized window.

- What might happen if we keep running the experiment (flipping a coin 20 times) again and again?
- What number(s) of heads would we expect to occur most often? Least often?

Now students are to experiment with different numbers of trials, such as 10, 20, 100, 200, and 500. Warn them that it may take time to simulate larger number of trials, but it is much faster than performing the experiment by hand!

Since the graphing calculator cannot store more than 999 elements in a list, the maximum number of trials is 999. Keep the number of flips the same (20).

Discuss the shape of the graph and how it changes as the number of trials increases.

Experiment 2

Students will now simulate a different binomial experiment—rolling a die 30 times and counting the number of 2s—using the program **DICEROLL**. They need to press <u>PRGM</u> to open the Program menu and choose it from the list.

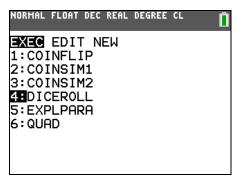
When prompted, students input the number of rolls, 30, and the number of trials, 1. After comparing with their classmates, they need to run the program again by pressing ENTER. This time, simulate repeating the experiment (rolling a die 30 times) 5 times.

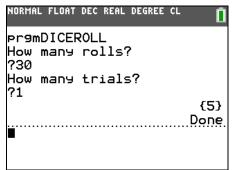
Ask student, *Do the results make sense?* They should expect numbers close to 5 because that's one-sixth of 30.

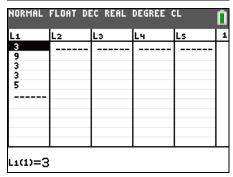
The **DICEROLL** program stores the results of each simulation in list **L1**. View the results there.

View the data in L1 it as a histogram.

Plot1 is already set to make a histogram of the data in **L1**. Simply press **Z00M** and select **ZoomStat** to view the plot of the new data an appropriately sized window.







Discussion Questions

- What might happen if we keep running the experiment (rolling a die 30 times) again and again?
- What number(s) of 2s would we expect to occur most often? Least often?

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Students use the **DICEROLL** program to experiment with different numbers of trials, such as 10, 20, 100, 200, and 500. Remind them that the maximum number of trials is 999 and to keep the number of rolls the same (30).

Discuss the shape of the graph and how it changes as the number of trials increases. Students should also compare it with the graph in Experiment 1.

Experiment 3

Finally, students simulate tossing a coin 10 times and counting the number of heads using the program **COINFLIP**. Once again, they will experiment with different numbers of trials, such as 10, 20, 100, 200, and 500, keeping the number of flips the same (10).

Discuss the shape of the graph and how it changes as the number of trials increases.

Students should notice that as the number of trials increases, the shape of the graph begins to resemble that of the graph from Experiment 1. This is the same discovery that statisticians made years ago: that as the number of trials increase, all binomial distributions begin to resemble each other.

Problem 2 - Probability Distributions

As the number of trials increases, the histogram approaches a bell-shaped curve called a **normal curve**. The **normal probability distribution** calculates probabilities based on this curve.

Explain to students that just as you can use different functions to model a set of x-y pairs, you can use different distributions to model a set of (outcome, P(outcome)) pairs. Just as there are many different families of functions (linear, quadratic, rational), there are many different families of probability distributions, and the best one must be chosen to model each situation.

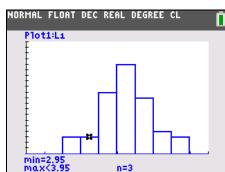
Students may be surprised to know that they have already worked with the **discrete uniform distribution**, which assigns the same (uniform) probability to each one of a set of events. An example of the discrete uniform distribution is rolling a die one time and looks like this:

$$\left\{ \left(1, \frac{1}{6}\right), \left(2, \frac{1}{6}\right), \left(3, \frac{1}{6}\right), \left(4, \frac{1}{6}\right), \left(5, \frac{1}{6}\right), \left(6, \frac{1}{6}\right) \right\}$$

Discuss other examples of the discrete uniform distribution and why it would be a not be a good choice to model Experiments 1, 2, or 3. You may also wish to discuss discrete versus continuous distributions. Experiment 3 will be used to explore these distributions.

Students need to simulate 50 trials of the experiment of flipping a coin 10 times.

View the results as a histogram. Press TRACE to see *n*, how many times each number of heads occurred. Record the results on your worksheet and use them to calculate the experimental probability of each number of heads.



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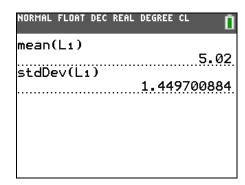
Different probability distributions are described in different ways. Again, draw the analogy to families of functions. A linear function is described by its slope and intercept, while a rational function is described by its asymptotes.

The binomial distribution is described by the probability of success and the number of trials per experiment (i.e. 10 in this case). The normal distribution is described by the mean and standard deviation of the data modeled.

To calculate the mean and standard deviation of the data in L1, students enter the commands **mean(** and **stdDev(** as shown at the right.

Press 2nd [LIST] and arrow to the MATH menu to access the commands.

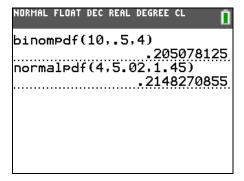
Press [2nd] [L1] to enter L1.



To find the theoretical probability of flipping exactly 4 heads in 10 coin flips predicted by each distribution, students will use the **binomPdf(** and **normalPdf(** commands from the DISTR menu (2nd [DISTR]).

binomPdf(number of flips per trial, probability of success, outcome)

normalPdf(outcome, mean, standard deviation)



Discussion Questions

- Which would you expect to be a closer match to experimental probabilities calculated from performing the experiment many times?
- Which would you expect to be a closer match to experimental probabilities calculated from performing the experiment only a few times?

Students need to rerun the simulation with more and fewer trials as described on their worksheet. Recalculate the theoretical probabilities with each distribution, noting that the mean and standard deviation have changed.

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Problem 3 - Application of the Normal Distribution

Explain to students that the distribution that is the best model depends on the situation. The normal distribution can model many situations that the binomial distribution cannot. The example given on the worksheet is one example.

Students are to graph the normal distribution described in this problem simply by entering the **normPdf** command in **Y1** with the variable **X** as the first argument, followed by the mean and the standard deviation. They will need to choose an appropriate viewing window for the graph.

 $10 \le x \le 20, -0.15 \le y \le 0.5.$

Then they are to calculate, on the Home screen, the probability that a randomly chosen student is 16. Students can confirm that it agrees with the graph of the normal distribution by graphing the probability as a horizontal line and finding its intersection with the curve. To find the intersection, press 2nd [CALC], choose **intersect** from the list, and follow the prompts.

