Mean & Median Smoothing

Student Activity

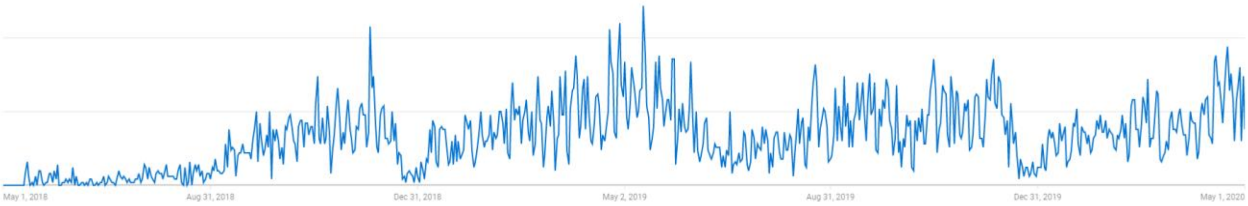
7 8 9 10 **11** 12

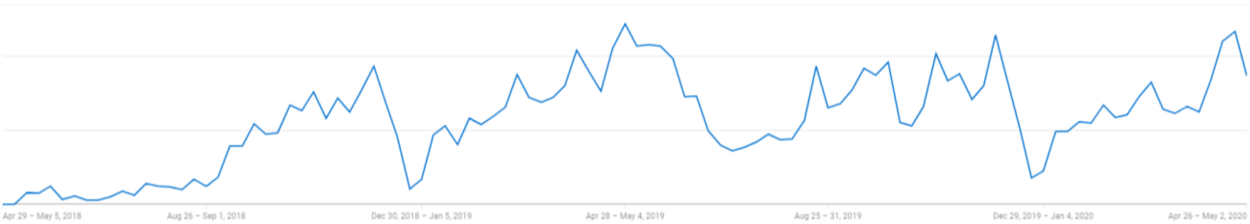
TI-Nspire™ Investigation Student 45 min

Introduction – From YouTube to Photoshop

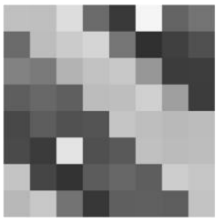
The charts below show the number of times a video on TI-Nspire was viewed on YouTube over a period of two years since the video was originally uploaded (May 2018). The top graph shows daily views and displays a lot of 'noise' in the form of daily fluctuations. To help identify general trends the second graph shows weekly views. The weekly view is a method of smoothing the data, effectively averaging out the views over a 7 day period.

Both charts show that the number of views is lower from December through to January; the second chart really shows how rapid the decline is from the start of December. As the content of the video is related to high school mathematics, the drop over the December - January period is not entirely unexpected. It is interesting to note that the weekly data seems to display a significant point in time when the views decline. This decline aligns remarkably well to the end of Year 11 and 12 studies each year. What other dips are evident? What is your forecast for June – July 2020?





Median smoothing is a method used by photo-imaging suites to automatically touch up a picture. The simplest way to understand this process is to consider a grey-scale image. Every pixel is represented by a single number between 0 (black) and 255 (white). The 300 x 400 pixel image below (left) has deteriorated with age. The image in the middle is an enlarged portion of the top left corner of the deteriorated image; immediately below are the numbers that generate this section. A pixel in the top row is almost white (245). To the left and right are the numbers 57 and 100. Below 245 are the numbers 118, 48 and 64. These numbers are deemed as being "immediately adjacent". The median of this data set: {48, 57, 64, 100, 118, 245} is (64 + 100) ÷ 2 = 82. The almost white pixel (245) will be replaced with dark grey (82).



192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
130	122	179	194	200	151	60	61
94	104	95	188	190	208	159	60
72	93	81	84	197	190	185	188
74	59	228	80	89	192	187	189
196	69	51	92	103	94	205	198
184	200	115	55	95	96	94	195

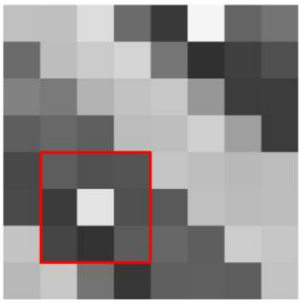
Author: P. Fox

Computer imaging software can scan every pixel and change each one based on the numbers immediately surrounding the pixel. The result is to correct all the drop outs in the deteriorated image with a view to returning the photo to its original glory, the photo shown on the right (previous page).

Question: 1.

Another pixel in the enlarged section is close to white (228) whilst the eight surrounding pixels are relatively dark: {93, 81, 84, 59, 80, 69, 51, 92} (see below). Determine the median value for the nine pixels and hence the new value for the 228 pixel.

192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
130	122	179	194	200	151	60	61
94	104	95	188	190	208	159	60
72	93	81	84	197	190	185	188
74	59	228	80	89	192	187	189
196	69	51	92	103	94	205	198
184	200	115	55	95	96	94	195



Question: 2.

Using the same set of pixels as the previous question, determine the replacement value for the 228 pixel if 'mean' smoothing was applied rather than median.

In the absence of filtering techniques that identify 'bad' pixels, every pixel would naturally undergo median or mean filtering, this has implications for the remaining image. This section of the image has a distinctive light and dark band. The following questions explore what happens to the image where the light band transitions to the dark band and vice-versa.

Question: 3.

Determine the result of median and mean smoothing on the centre (189) pixel and comment on the result.

192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
130	122	179	194	200	151	60	61
94	104	95	188	190	208	159	60
72	93	81	84	197	190	185	188
74	59	228	80	89	192	187	189
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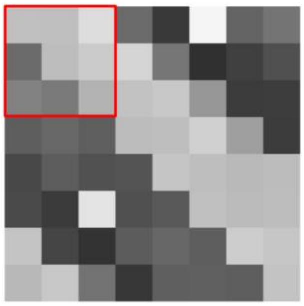

Question: 4.

Determine the result of median and mean smoothing on the centre (192) pixel and comment on the result.

192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
130	122	179	194	200	151	60	61
94	104	95	188	190	208	159	60
72	93	81	84	197	190	185	188
74	59	228	80	89	192	187	189
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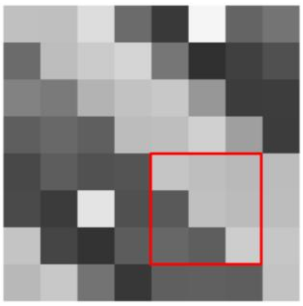

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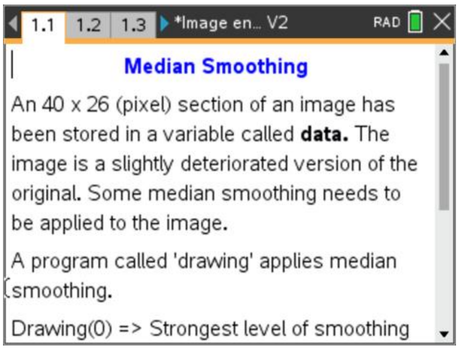
Automating the process

Open the TI-Nspire file: Image Enhancement



Page 1.1 contains a Notes Application providing some detail about the size of the image that has been embedded in this file. The image is a small section of the image shown on page 1 of this worksheet.

Read through the instructions on Page 1.1 and then navigate to Page 1.2.

Select the drawing program from the VAR menu:

drawing(100)

When the drawing program is launched from Page 1.2, two versions of the image will be displayed on the drawing canvas.

The image on the left is part of the original. The image on the right has been filtered, however by using: drawing(100), no filtering has actually been completed.

Can you identify which region of the original image has been captured?

To return to the calculator application where the program was launched, press [esc].

It is now time to allow median smoothing to repair the pixels. Run the Drawing program again as:

Drawing(0)

Question: 5.

Did the filtering tool remove all the drop-out pixels? What else changed about the image? Explain.

It would be nice if the data smoothing only applied to points that were clearly indicative of deterioration or pixel drop-out. The number used in the 'drawing' program provides such an opportunity. The program continues to run median smoothing, however the number determines just when it will happen. The number determines how many 'standard deviations' the current pixel is away from the mean of the surrounding pixels.

Question: 6.

To help understand how this parameter affects this selective filtering process we return to the set of pixels investigated in Question 3.

192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
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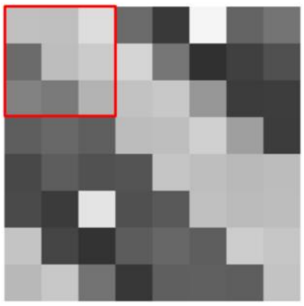

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a) Determine the mean and standard deviation of this set of numbers corresponding to the cluster of 9 pixels.

b) If drawing(10) is used, then the middle pixel remains unchanged if it falls within the following range:

$$\bar{x} - \sigma \leq x \leq \bar{x} + \sigma$$

Use your answers to part (a) to determine whether or not 189 is outside this range and therefore the value of this pixel once this form of selective filtering is completed.

Question: 7.

The region studied in Question 1 contains a pixel that dropped out.

192	191	221	107	57	245	100	116
107	189	203	213	118	48	64	80
130	122	179	194	200	151	60	61
94	104	95	188	190	208	159	60
72	93	81	84	197	190	185	188
74	59	228	80	89	192	187	189
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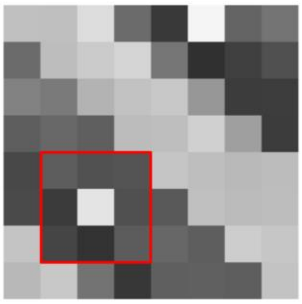

a) Determine the mean and standard deviation of this set of numbers corresponding to the cluster of 9 pixels.

b) If drawing(10) is used, then the middle pixel remains unchanged if it falls within the following range:

$$\bar{x} - \sigma \leq x \leq \bar{x} + \sigma$$

Use your answers to part (a) to determine whether or not 228 is outside this range and therefore the value of this pixel once this form of selective filtering is completed.

Question: 8.

Explain why the standard deviation might be used as a guiding statistic rather than Interquartile range.

Question: 9.

Explain why the deviation from the 'mean' rather than the 'median' might be used in this refined process.

Question: 10.

Run the filtering tool again using: drawing(10). Compare the original image (with pixel drop-outs) to the filtered image. Did the filtering still remove the drop-outs? (Explain) What other differences can you see when this added restriction is placed on the filtering.

Question: 11.

Run the filtering tool again using: drawing(20). Once again compare the original image (with pixel drop-outs) to the filtered image. What differences do you notice this time?

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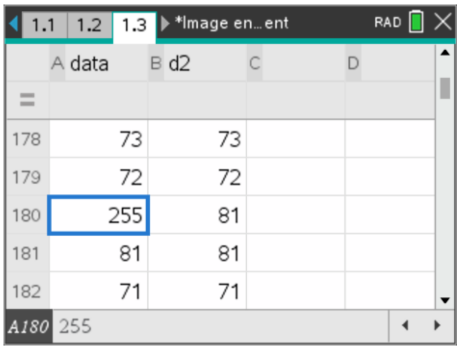
Further Deterioration

What happens if our image has deteriorated further? If there are more pixel dropouts 'clusters' of dropouts become increasingly more likely. The existing dropouts are in the following locations:

Pixel: 180

Pixel: 495

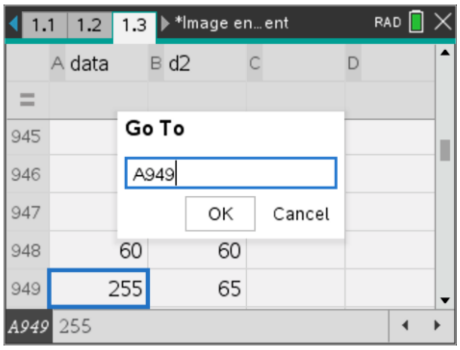
Pixel: 910



Adding dropouts adjacent to these existing locations makes it possible to see how clustering affects the filtering process.

The following pixels changes need to be made to the image 'data'.

Pixel: 181	Change to:	255
Pixel: 535		255
Pixel: 949		255



To navigate to a specific cell in the spreadsheet: Press Ctrl + G and enter the cell reference. For example entering: **A949** would set the current cell to one of the pixels that needs to be altered.

Question: 12.

With the increased 'damage' to the original image, run the filtering tool again using: drawing(20).

a) Did all the pixel drop-outs get fixed? Explain.

b) Try running the program again as: drawing(10). Did all the pixels get fixed, if so, at what 'cost'?

c) Try using other drawing values such as 17 or 15. What value removes the dropout pixels with minimal compromise to the quality of the remaining image?

Question: 13.

Suppose a new median filtering tool was created that sampled an additional layer of numbers around any given pixel. (Sample of 25). Discuss how this might affect filtering and image quality.

Question: 14.

Design a new statistically based filtering tool that might help eliminate larger 'damaged' regions of an image without losing other aspects of the image quality.

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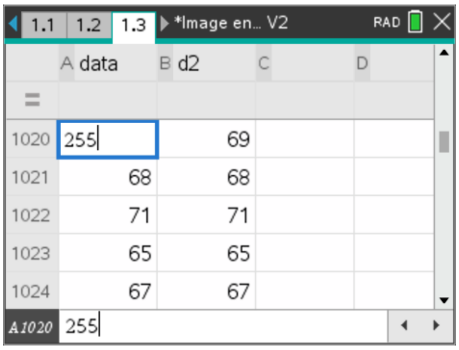
d) It's now time to 'damage' the original image. Single pixels will be replaced with either white or black pixels.

e) [Read the Warning message and Tip before completing this step.]

f) Navigate to Page 1.3. This page contains a Spreadsheet application. The 'data' list represents all the pixels from the image. Your first task is to add 6 individual drop-out pixels randomly through data. The pixels should be either white (255) or black (0). Do not put the pixels too close together; allow at least 80 pixels between your drop-outs.

g)



Record each of your changes! The original number and its location. List D2 contains the smoothed data.

To navigate quickly up and down the spreadsheet use Ctrl + 9 to move a page up or Ctrl + 3 to move a page down at a time. Ctrl + 7 navigates to the top of the list and Ctrl + 1 to the bottom. Once the cell is selected simply type the new entry, 0 or 255, and the old entry will automatically be replaced.

The filtered pixels are sent to: **d2**. The **data** list is not changed during or after the filtering process. If you wish to return the **data** list to its original values, in the Calculator Application, type: data:=backup

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Correcting colour images is a slightly slower process as each pixel is represented by three numbers:

Monitor Display (light):	Red: 0 – 255	Green: 0 – 255	Blue: 0 – 255
Photograph (Print):	Magenta: 0 - 255	Yellow: 0 - 255	Cyan: 0 - 255

The down side to median correction is that the regions between two contrasting objects can be broken down leaving a less sharp image. To correct this problem, other mathematical algorithms are used!

The determination of the median of a set of numbers is a two-step process:

Rank the numbers in ascending order

Locate the middle number

When you graph a set of numbers the first step is automatically resolved. In this activity you will work through four data sets to see how median smoothing can be used to remove noise from data or a graph of the data.

Activity …

Open the TI-Nspire file …

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