Polar Conics

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will understand that the equations for conics can be expressed in polar form.
- Students will be able to describe the relationship between eccentricity and the type of conic section.
- Students will be able to describe the relationship between distance from the conic to the directrix and the graph of the conic section.
- Students will be able to describe the effects of a phase shift in the polar form of an equation for a conic on the graph of the conic.
- Students will look for and make use of structure (CCSSM Mathematical Practice).

Vocabulary

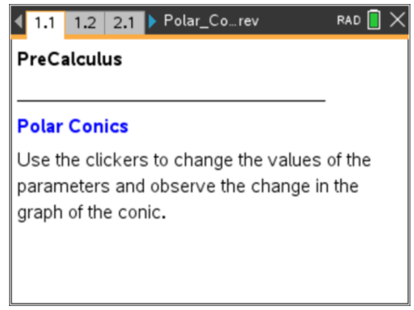
- polar form
- conic
- parabola
- ellipse
- hyperbola
- directrix
- eccentricity
- phase shift

About the Lesson

- This lesson involves exploration of polar equations for conic sections.
- As a result, students will:
  - Manipulate the parameters of polar equations for conics and observe the results.
  - Investigate the parameters required to produce a specified conic.

TI-Nspire™ Navigator™ System

- Transfer a File.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Polar_Conics_Student.pdf

Polar_Conics_Student.doc

TI-Nspire document

Polar_Conics.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

A conic is defined as the locus of points in a plane whose distance from a fixed point (focus) and a fixed line (directrix) is a constant ratio. This ratio is called the eccentricity, *e*, of the conic. The polar notation for the ellipse, hyperbola, and parabola is given by the equation:

$$r = \frac{ed}{1 \pm e\cos(\theta)} \text{ OR } r = \frac{ed}{1 \pm e\sin(\theta)}$$

where *e* is the eccentricity and *d* is the distance from the origin to the directrix.

By expressing the equation in polar coordinates, we can generate all three types of conics from a single equation.

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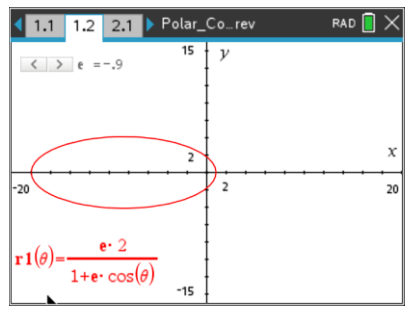

1. Use the clicker to change the values of the eccentricity, *e*. For what values of *e* is the conic a parabola? An ellipse? A hyperbola?

   Sample Answers: When *e* = 1, the conic is a parabola. When |*e*| < 1, the conic is an ellipse. When |*e*| > 1, the conic is a hyperbola.

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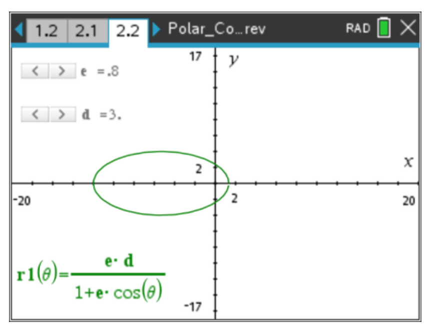

2. Use the clicker to change the values of *d*, the distance between a point on the conic and the directrix.
   a. Set *e* = 1. When the conic is a parabola, what effect does *d* have on the graph of the function?

   Sample Answers: The larger *d* is in magnitude, the wider the parabola is. When *d* is positive, the parabola opens to the left, and when *d* is negative, the parabola opens to the right.

b. Set *e* < 1. When the conic is an ellipse, what effect does *d* have on the graph of the function?

Sample Answers: As *d* increases in magnitude, the ellipse grows larger. When *d* is positive, the center of the ellipse is on the negative x-axis. When *d* is negative, the center of the ellipse is on the positive x-axis.

c. When the conic is a hyperbola, what effect does *d* have on the graph of the function ?

Sample Answers: When *d* is very small in magnitude, the vertices of the branches of the hyperbola are very close to each other. When *d* is larger in magnitude, the vertices of the branches of the hyperbola are farther apart. When *d* is positive, increasing d shifts the hyperbola to the right. When *d* is negative, decreasing *d* shifts the hyperbola to the left.

3. Adjust the parameters to create an ellipse that is 9 units in width, and make a note of those parameters. Are these the only parameters that will create such an ellipse? Explain.

Sample Answers: For example, when *e* = .8 and *d* = 2, the ellipse is approximately 9 units wide. There are other parameters that will create such an ellipse, for example, when *e* = -.8 and *d* = 2.

4. Adjust the parameters to create a hyperbola for which the vertices of the branches are 6 units apart, and make a note of those parameters. Are these the only parameters that will create such a hyperbola? Explain.

Sample Answers: For example, when *e* = 1.4 and *d* = 2, the vertices of the hyperbola branches are 6 units apart. There are other parameters that will create such a hyperbola, including *e* = 1.6 and *d* = 3.

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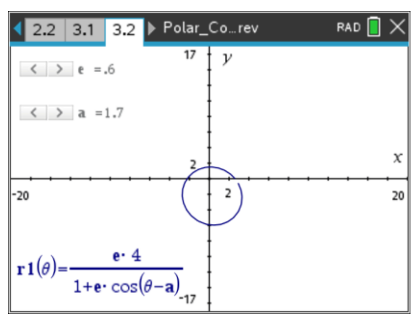

5. Use the clicker to adjust the value of *a*, the phase shift.

a. Set *e* = 1. When the conic is a parabola, what effect does *a* have on the graph of the function?

Sample Answers: When the conic is a parabola increasing, *a* rotates the parabola in the counter-clockwise direction.

b. Set *e* < 1. When the conic is an ellipse, what effect does *a* have on the graph of the function?

Sample Answers: Increasing *a* rotates the ellipse counter-clockwise.

c. Set $e > 1$. When the conic is a hyperbola, what effect does a have on the graph of the function?

Sample Answers: Increasing *a* rotates the hyperbola counter-clockwise.

6. Is it possible to adjust the values of *a* and *e* so that the resulting conic is a parabola centered about the y-axis? If so, what parameters yield this result? If not, explain why not.

Sample Answers: Yes, for example, when $e = 1$ and $a = 1.6$, the parabola appears to be centered about the *y*-axis.

7. Which type of conic will result from each of the following equations? How do you know?

a. $r = \frac{10}{1+3\cos(\theta-5)}$

Sample Answers: The graph is a hyperbola, because $e = 3$. The distance between a point on the hyperbola and the directrix is 10/3, so the vertices of the parabolas branches will be farther apart. There is a phase shift, so the hyperbola will not be centered about the x-axis.

b. $r = \frac{3}{1-\cos(\theta-6)}$

Sample Answers: The graph will be a parabola because $e = -1$. The distance between a point on the parabola and the directrix is 3, so the parabola will be wider. There is a phase shift, so the parabola will not be centered about the x-axis.

c. $r = \frac{20}{1-0.5\cos(\theta-2)}$

Sample Answers: The graph will be an ellipse because $e = -0.5$. The distance between a point on the ellipse and the directrix is $20/0.5 = 40$, so the ellipse will be very large. There is a phase shift, so the ellipse will not be centered about the x-axis.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- That one polar equation can be used to express all three types of conics.
- The effects eccentricity has on the conic formed.
- The effects of distance to the directrix on the size of the conic.
- The effects of phase shift on the orientation of a conic.

Assessment

1. Teachers might want to give students additional equations in polar form and ask students to describe the conics.
2. If students have experience with conic equations in Cartesian form, teachers might want to have students match Cartesian and polar equations of conics.
3. Teachers might also want to give students specifications for a conic and have students generate equations for the conic or match to equations provided by the teacher.