Case File 14

Hot Air, Cold Body: Using Newton's law of cooling to determine time of death

Use Newton's law of cooling to narrow down the number of suspects by determining when the victim was killed.

Memo to Detective Sergeant:

The elevator operator of the Ritz Palace Hotel died from a stab wound while on duty last Thursday evening. His body was discovered by a family on its way down to the pool. When we arrived at the scene, we canvassed the area but found nothing. The elevator is full of fingerprints of the hundreds of guests who ride it during the day. We have several suspects in mind, but we are having trouble pinning down the time of death. If we can determine that, we have a good shot at finding the killer.

Enclosed are a photograph of the crime scene and part of the paramedic report.

Paramedic report

Date: 10/5/05

Time: 9:45 p.m.

Body temperature: 29.0°C

Notes: Elevator temperature was high; thermostat set at 27°C.

Forensics Objective

- determine the time of death of a person who has died within the last few hours

Science and Mathematics Objectives

- create a temperature vs. time graph for cooling
- use the cooling-rate equation to estimate time of death
- become familiar with Newton's law of cooling

Materials

- TI-83/TI-84 Plus™ Family
- Vernier EasyTemp™ temperature probe
- Vernier EasyData™ application
- ring stand with clamp
- model victim

Procedure

Part I: Collecting the Data

1. Plug the temperature probe into the USB port on the calculator. The calculator will turn on automatically, and the EasyData App will display the ambient (room) temperature. At the bottom of the screen, just above the menu buttons, the current experimental setup will be displayed.

TIP! At the bottom of the Main screen are five options (**File**, **Setup**, **Start**, **Graph**, and **Quit**). Each of these options can be selected by pressing the calculator key located below it (Y=, WINDOW, ZOOM, TRACE, or GRAPH).

2. The default experimental setup for the EasyTemp probe is to collect one sample every second for 3 minutes. However, for this experiment, you will need to collect one data point every 10 seconds for 20 minutes. Change the experimental setup so the calculator will collect 120 samples at 10-second intervals for a total experiment time of 1200 seconds.
   a) Select **Setup** from the Main screen.
   b) Select option **2: Time Graph…**.
   c) Select **Edit** to change the sample interval.
   d) Press CLEAR and then type **10** as the sample interval.
   e) Select **Next** to change the number of samples.
   f) Press CLEAR and then type **120** as the number of samples.
   g) Select **Next** to confirm that the experimental setup is correct.
   h) When you have confirmed that the time graph settings are correct (10-second sample interval, 120 samples, 1200-second experiment length), select **OK**.

3. Read the ambient (room) temperature from the display on the Main screen and write it in your Evidence Record. Make sure the tip of the probe is not touching anything warmer or cooler than room temperature (such as your hand or the tabletop).

4. Place the tip of the EasyTemp probe into the model victim. Use the ring stand and clamp to hold the probe in place.

5. Select [Start] to begin collecting data. If you get a message about overwriting stored data, select [OK]. Data collection will run for 20 minutes. During the data collection, a graph of temperature vs. time will be displayed. When the data collection is complete, the temperature vs. time graph will be scaled and displayed, along with several options for working with the graph. If the screen goes blank during or after the data collection, press [ON] to restore it.

6. When data collection is complete, use the arrow keys to "trace" your temperature vs. time graph. The cursor will move along the curve. The *x*-value (time in seconds) will be displayed at the top of the screen next to **X=**. The *y*-value (temperature in degrees Celsius) will be displayed next to **Y=**.
   a) Trace the graph to locate the maximum temperature reached by the model victim during the data collection. Record this temperature as the initial temperature of the model in your Evidence Record. (Note: The maximum temperature will probably *not* be the first temperature reading on your graph; be sure to trace the graph to find the *maximum* temperature).
   b) In your Evidence Record, write the time (in seconds) at which the maximum temperature occurred. This is the initial time.
   c) Locate the minimum temperature reached by the model, and record it in your Evidence Record as the final temperature. Record the time it occurred as the time of minimum model temperature. This is the final time. (Note: The minimum temperature should be the *last* point on your graph).
   d) Subtract the initial time from the final time to find how long your model victim was cooling. Enter this time in your Evidence Record as the duration of model temperature measurement.

7. Before removing the temperature probe, exit the EasyData App.
   a) Select [Main] to return to the Main screen.
   b) Select [Quit] then [OK] to return to the Home screen.
   c) You can now remove the temperature probe.

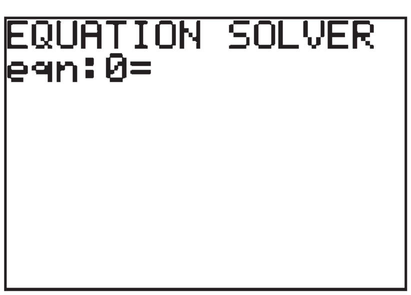
Part II: Analyzing the Data

To use your data to determine time of death, you can use Newton's law of cooling:

$$\text{cooling time} = \left(\frac{1}{k}\right)\ln\left(\frac{\text{initial temperature} - \text{ambient temperature}}{\text{final temperature} - \text{ambient temperature}}\right)$$

where *k* is the cooling constant. You can use your calculator to solve this equation for different variables. First you will solve the equation for *k*. Then you will use this value of *k* to determine an estimate for how long the body was cooling before its temperature was measured.

8. First, enter the equation for Newton's law of cooling into your calculator's equation solver.
   a) From the Home screen, press [MATH]. Select option **0: Solver...**. You should see a screen that looks like this:



   If the **eqn:0=** line is not blank, use the [▲] key to move to the **eqn:0=** line and press [CLEAR] to clear it.

b) In order for the calculator to solve the equation for you, the left side of the equation must be equal to 0. To rewrite the Newton's law of cooling equation so that the left side is 0, subtract "cooling time" from both sides of the equation. This will leave you with the following equation:

$$0=\left(\frac{1}{k}\right)\ln\left(\frac{\text{initial temperature}-\text{ambient temperature}}{\text{final temperature}-\text{ambient temperature}}\right)-\text{cooling time}$$

c) You will need to use letters to represent each variable in the equation. Use **K** for *k*, **I** for initial temperature, **A** for ambient temperature, **F** for final temperature, and **T** for cooling time. With these letters substituted for the different variables, the equation looks like this:

$$0=\left(\frac{1}{\mathrm{K}}\right)\ln\left(\frac{\mathrm{I}-\mathrm{A}}{\mathrm{F}-\mathrm{A}}\right)-\mathrm{T}$$

d) You will need to type the Newton's law of cooling equation into the equation solver. Remember that you can type letters onto the screen by pressing [ALPHA] and then pressing the calculator key that has the letter you want written above it in green. For example, the letter **K** is inserted by pressing [ALPHA] [(], and the letter **T** is inserted by pressing [ALPHA] [4].

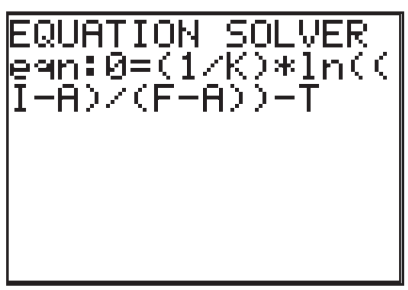
e) Now begin entering the equation into the calculator. Type [(] [1] [÷] [ALPHA] [(] [)] to insert **(1/K)**. This inserts the first part of the Newton's law of cooling equation. Do *not* press [ENTER].

f) Type [X] [LN] to enter the natural logarithm, ln, function. Do *not* press [ENTER]. Notice that a left parenthesis is automatically inserted when you insert the natural log function. You will have to close the parentheses when you are finished entering the equation.

g) Press [(] [ALPHA] [x^2] [−] [ALPHA] [MATH] [)] to enter the numerator of the equation inside the natural logarithm. Do *not* press [ENTER].

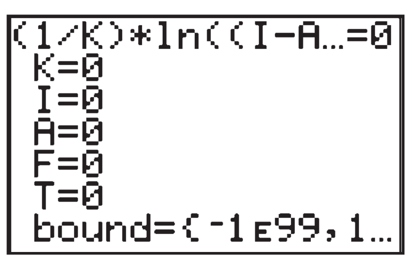
h) Type [÷] [(] [ALPHA] [COS] [−] [ALPHA] [MATH] [)] [)] to enter the denominator and close the parentheses around the natural logarithm. Do *not* press [ENTER].

i) Type [−] [ALPHA] [4] to enter **⁻T**. Your screen should now look like this:



Notice that the equation you have just entered looks like the equation from step 8c.

j) Press [ENTER]. Your screen should give the equation and then a list of the five variables: **K**, **I**, **A**, **F**, and **T**. The screen should look like this:



(Note: Don't worry if the values next to the variables are different from those in the screen shot. You're going to change them in the next step anyway.)

9. Now use the solver function to calculate the value of *k*, the cooling constant.

> If you make a mistake or if there are already values assigned to the variables, use the arrow keys to move to the variable and then type in the correct value.
>
> TIP!

a) In order for the calculator to solve for *k*, you need to give it a starting "guess" for the value. In most cases, *k* is approximately 0.0001. Enter **0.0001** for the **K** variable. Press ENTER to move to the next variable.

b) Recall that variable **I** stands for the initial temperature of the object that is cooling. For variable **I**, type in the initial temperature for the model that you measured. This is the *maximum* temperature that you recorded from your temperature graph in step 6. Press ENTER to move to the next variable. (Note: Type in *only* the number values for your temperatures. Do not type in º, C, or any other symbols. For 23ºC, simply type in **23**.)

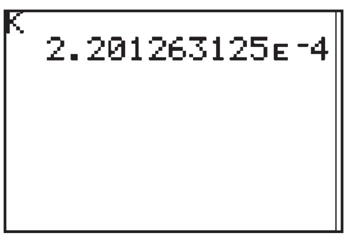
c) Variable **A** stands for the ambient (room) temperature around the object that is cooling. For variable **A**, type in the ambient air temperature that you recorded in step 3. Press ENTER to move to the next variable.

d) For variable **F** (the final temperature of the cooling object), type in the final (minimum) temperature of the model. Press ENTER to move to the next variable.

e) For variable **T** (the cooling time), type in the duration of the model temperature measurement (in seconds) that you calculated in step 6d (it should be no longer than 1200 seconds).

f) The calculator will solve the equation for whichever variable is highlighted. Since you want to solve for *k*, use the arrow keys to move the cursor to highlight the number next to **K**. Do *not* press ENTER.

g) Press ALPHA ENTER to select the **SOLVE** function and solve the equation for **K**. (Note: It may take a few seconds for the calculator to solve for **K**. Be patient!) The **K** value will probably be a very long decimal number followed by **....** In order to see the value of this number more clearly, go back to the Home screen by pressing 2nd MODE. Type **K** by pressing ALPHA (. Then press ENTER to display the value of **K**. For example, in the sample screen below, **K** is equal to 2.20×10^{-4}.



h) Write the value of *k* in your Evidence Record. Record three significant figures.

10. Now that you know the cooling constant, you can use the solver function to determine how long the body cooled before the paramedics measured its temperature. To figure this out, you need to solve the cooling equation for **T**, cooling time.

a) Press MATH 0 to go back to the equation solver screen.

b) Leave the **K** variable set to its current value. Press ENTER to move to the **I** variable. In this case, the body cooled from normal body temperature (37ºC) to its temperature when it was measured. The initial temperature was therefore 37ºC. Record this in the Evidence Record, then type **37** for **I** in the calculator and press ENTER to move to the next variable.

c) The ambient temperature, **A**, should be set to the temperature of the elevator the body was found in. Check the evidence report to find this temperature. Write it in the Evidence Record and then enter it into variable **A**. Press ENTER to move to the next variable.

d) The final temperature, **F**, is the temperature of the body when it was measured by the paramedics. Check the evidence report for this temperature, record it in your Evidence Record, and then type it into the space for the **F** variable. Press ENTER to move to the next variable.

e) In this case, you are trying to solve for **T**, which is the length of time the body was cooling before the paramedics measured its temperature. Press CLEAR to clear the value of **T**, and enter a “guess” of 1800 seconds for the cooling time. (The body was probably in the elevator for longer than 1800 seconds, which is about half an hour, but the guess you give the calculator does not have to be completely accurate).

f) Leave the cursor on the **T** variable and press ALPHA ENTER to solve the equation for **T**.

g) Record the value of *t* in seconds in your Evidence Record.

h) Convert the cooling time from seconds to minutes by dividing it by 60. Write the cooling time in minutes in your Evidence Record.

11. To determine time of death, subtract the number of minutes that the body was cooling from the time that the temperature was measured. Enter the time of death into the Evidence Record.

Name: ____
Date: ____

Evidence Record

From the Model

Ambient (room) temperature for model (°C) ____________________

Initial temperature of model (°C) ____________________

Final temperature of model (°C) ____________________

Time of maximum model temperature (s) ____________________

Time of minimum model temperature (s) ____________________

Duration of model temperature measurement (s) ____________________

Cooling constant, *k* ____________________

From the Evidence Report

Ambient (room) temperature for body (°C) ____________________

Time body temperature was measured ____________________

Temperature of body (°C) ____________________

Cooling time, *t* (s) ____________________

Cooling time, *t* (min) ____________________

Actual time of death ____________________

Case Report

Prepare a report stating the conclusions of your investigation. In your report, be sure to include the temperature measurements that you made, all of your calculations, and the estimated time of death you calculated. How accurate do you think your time-of-death estimate is? What factors can exist that may make your time-of-death estimate inaccurate? What factors can exist that may make a *real* time-of-death estimate inaccurate?

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Teacher Notes

Teaching time: one class period

This lab introduces a practical use for Newton's law of cooling and the equation that describes it.

Tips

- You'll need to practice the lab to find out what the cooling rates are like in your room.
- There will be approximately 20 minutes during which the calculator will be collecting temperature data but the students will not actively be doing anything. This may be a good time to review Newton's law of cooling or explain some background information.
- If you wish, you can have each group's model "victim" start at a different body temperature by heating the canisters in water baths of different temperatures.

Lab Preparation

Materials for making the model victim
- sodium polyacrylate (water lock; can be purchased through chemical supply companies)
- Gatorade® or water
- 35 mm film canister, with hole in lid the size of temperature probe tip

Instructions for making the model victim
1. Place 1 g sodium polyacrylate in an empty film canister.
2. Fill the canister three-fourths full of Gatorade or water.
3. Put the cap on the canister and shake it gently, making sure to cover the hole.
4. Warm the canister in your hands. Alternatively, you can place the canister in a water bath set to the desired temperature.

Background Information

Body temperature readings for actual time-of-death estimates are usually taken rectally or intra-abdominally (in the liver). Ear, mouth, or armpit temperatures are generally considered inaccurate for these measurements. Depending on the level of your students, you may or may not wish to mention this or explore it further. In real life, the rate of cooling depends upon many things. The size of the body, how the body is dressed, the room temperature, where the corpse is located, and humidity all affect how fast a corpse cools. For this reason, investigators often use several different methods to estimate time of death. You should emphasize to your students that the model they are using to determine the cooling constant is just that—*a model*.

Resources

http://www.dundee.ac.uk/forensicmedicine/llb/timedeath.htm
This site from the University of Dundee (UK) includes interesting quotations on the difficulties in determining time of death, as well as a thorough exploration of the many changes that occur in the body after death.

http://www.pathguy.com/TimeDead.htm
This site contains a Java-based interactive time-of-death calculator. It also includes links to several other good forensic pathology sources.

Modifications

- You can ask more-advanced students to investigate other methods of determining time of death (e.g., forensic entomology). However, be aware that many resources and methods may involve rather gruesome pictures, descriptions, or diagrams. You can also introduce and discuss some of the major sources of error in estimating time of death from body temperature. For example, what would happen to the time-of-death estimate if the person had a fever when he or she died? What if the person died of hypothermia?
- Students who are less comfortable or experienced using the calculator can solve the cooling-rate equation by hand, without using the calculator's equation solver function. To do this, use the following information and steps for Procedure Part II:

  To use your data to determine time of death, you can use Newton's law of cooling, as follows:

$$\text{cooling time} = \left(\frac{1}{k}\right)\ln\left(\frac{\text{initial temperature} - \text{ambient temperature}}{\text{final temperature} - \text{ambient temperature}}\right)$$

  where *k* is the cooling constant. First, you will rearrange the equation to solve for k for the temperature measurements that you took with the model. Then, you will solve for cooling time, using the equation and the measurements that the paramedics took of the victim. Once you know how long the body was cooling before it was measured by the paramedics, you can figure out what time the victim died.

8. In order to solve for *k*, you need to rearrange the Newton's law of cooling equation as follows:

$$k = \frac{1}{\text{cooling time}}\ln\left(\frac{\text{initial temperature of model} - \text{ambient temperature}}{\text{final temperature of model} - \text{ambient temperature}}\right)$$

   Using the values for time and temperature that you measured for the model, solve the equation above to determine the value of *k*. Record this value in your Evidence Record.

9. Now you can use the Newton's law of cooling equation to solve for the cooling time.

$$\text{cooling time} = \frac{1}{k}\ln\left(\frac{\text{normal body temperature (37°C)} - \text{ambient temperature}}{\text{temperature of body when found} - \text{ambient temperature}}\right)$$

   a) Look at the evidence report to learn the temperature of the body when it was measured and the ambient temperature of the elevator. Write these values in your Evidence Record.
   b) Solve the equation above for cooling time, using the value of k that you calculated in step 8 and the temperatures you just recorded from the evidence report. The cooling time that you get will be in seconds. Record the cooling time in seconds in your Evidence Record.
   c) Convert the cooling time from seconds to minutes by dividing it by 60. Record the cooling time in minutes in your Evidence Record.

10. To determine the time of death, subtract the number of minutes that the body was cooling from the time it was measured. Record the time of death in your Evidence Record.

Sample Data (using Gatorade)

From the Model

Ambient (room) temperature for model (°C)	23.0°C
Initial temperature of model (°C)	28.6°C
Final temperature of model (°C)	27.3°C
Time of maximum model temperature (s)	0 s
Time of minimum model temperature (s)	1200 s
Duration of model temperature measurement (s)	1200 s
Cooling constant, *k*	2.20×10^{-4}

From the Evidence Record

Ambient (room) temperature for body (°C)	27°C
Time body temperature was measured	9:45 p.m.
Temperature of body (°C)	29.0°C
Cooling time, *t* (s)	7311 s
Cooling time, *t* (min)	122 min
Actual time of death	7:43 p.m.

Case Report Notes

In their case reports, students should indicate understanding of the various factors that could have affected their calculations. For example, the cooling constant for the model was probably not the same as the cooling constant for the body. In addition, temperature or time measurements could have been inaccurate (due to mistakes in reading the graph or to sampling frequency). The students should also indicate an understanding that many factors influence a real time-of-death estimate. These factors include the size and composition of the body, how the body is dressed, humidity, location of the body, whether the body has been touching anything very hot or cold, the person's body temperature before death, whether the body was moved, whether the ambient temperature was constant, and whether body temperature equaled ambient temperature.