Math Objectives

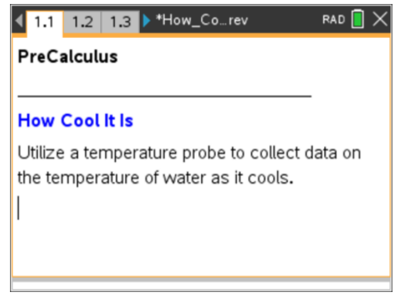
- Students will determine and analyze an exponential model for the temperature of water as it cools.
- Students will make predictions about the time required to reach various water temperatures and about the water temperature at various times.
- Students will apply mathematics to solve problems arising in everyday life, society, and the workplace (CCSS Mathematical Practice).
- Students will interpret mathematical results in the context of the situation and reflect on whether the results make sense, improving the model as necessary (CCSS Mathematical Practice).
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematical Practice).

Vocabulary

- exponential function
- regression equation
- sum of squares due to error

About the Lesson

- This lesson involves creating an exponential regression equation to model the temperature of water as it cools.
- As a result, students will:
  - Learn how to adjust their equation to produce an equation that will better model the data.
  - Adjust their equation to incorporate the fact that water will not get cooler than the temperature of the room.
  - Use sliders to change the values of the variables *a* and *b* in their model to produce the smallest SSE (the Sum of Squares due to Error) value.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing ctrl G. The entry line can also be expanded or collapsed by clicking the chevron.

Lesson Files: *Student Activity*
How_Cool_It_Is_Student.pdf
How_Cool_It_Is_Student.doc

TI-Nspire document
How_Cool_It_Is.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

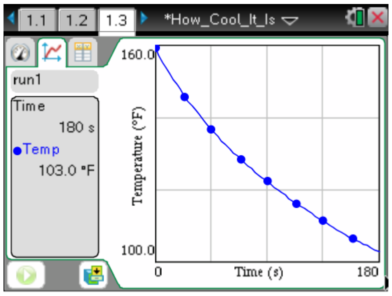
TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge and monitor students' understanding.
- Use Live Presenter to provide assistance to students throughout the activity.
- Use Quick Poll to assess students' understanding and compare equations.

Discussion Points and Possible Answers

Tech Tip: For this activity, you might want to put students into groups of two.

Move to page 1.3.



Connect the EasyTemp™ temperature probe to the mini-USB port of your TI-Nspire. Place the sensor in a cup of hot water, and leave it there for about 30 seconds.

Remove the temperature probe from the water, and rest it on the edge of a table. Do not let anything touch the tip of the probe. Click the green arrow in the bottom left-hand corner of the page to begin data collection.

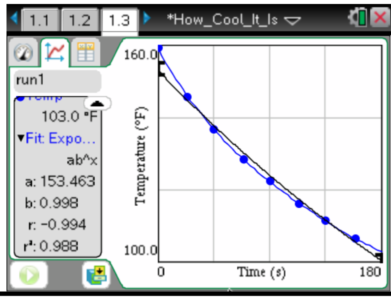
Data will be collected for three minutes. You can then disconnect the EasyTemp™ probe, and carefully clear away the hot water.

1. What type of function appears to be a good model for the data? Explain your reasoning.

   Sample Answers: The curve appears to be an exponential function because of its shape.

TI-Nspire Navigator Opportunity: *Screen Capture and Live Presenter*
See Note 1 at the end of this lesson.

2. To generate a regression equation, select **MENU > Analyze > Curve Fit**. Select a curve fit option, and press enter.



TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

a. Write the regression equation.

Sample Answers: Answers will vary, depending upon the data that the students collect. One possible answer is: $y = 153.463(0.998)^x$.

b. Do you think the regression equation will be a good model for t > 180 seconds? Why or why not?

Sample Answers: The regression equation cannot continue to model this function as time increases. The horizontal asymptote for the regression equation is y = 0, since as *x* gets larger and larger, 0.998^x gets smaller and smaller, and $153.463(0.998)^x$ also gets smaller and smaller. We can say that $\lim_{x\to\infty} 153.463(0.998)^x = 0.$
A hot liquid tends to cool down until it reaches room temperature and then remains constant. Roughly speaking, it is true that the temperature of the liquid decreases exponentially as the liquid cools, but this model does not continue to fit the data very well as we can see by looking at the graph.

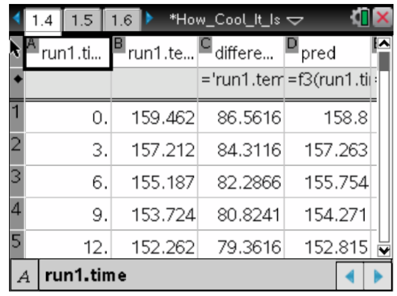
3. a. If we continued to take readings of the temperature of the water, what is the lowest temperature the water would reach?

Sample Answers: The water temperature will eventually reach room temperature.

b. Is this value consistent with the regression equation? Explain.

Sample Answers: This is not consistent with the regression equation, since that equation has a horizontal asymptote of y = 0. This indicates that the water temperature could reach a temperature close to 0°. Since the water temperature will never be less than room temperature, it must be modeled with an equation that has an asymptote of y = room temperature.

Move to page 1.4.



Columns A and B of the spreadsheet contain the data from the lab. Column C is the difference between the temperature of the water at time, x, and the room temperature.

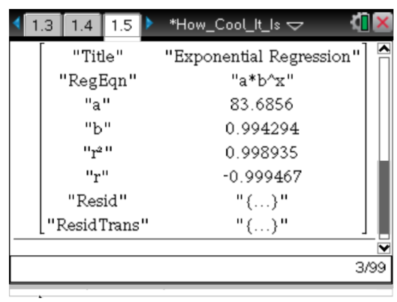
In the shaded formula cell for the **difference** column, type: = ctrl L. Choose "run1.temperature." Subtract the room temperature from "run1.temperature." Thus, this column now contains the values of the difference between the run1.temperature and room temperature, i.e.,

$$y_{difference} = y_{run1.temperature} - y_{room\ temperature}.$$

We can now obtain a regression equation for $y_{difference}$.

> **Tech Tip:** To determine the temperature of the room, connect the temperature probe to the handheld and wait until the temperature levels off.

Move to page 1.5.

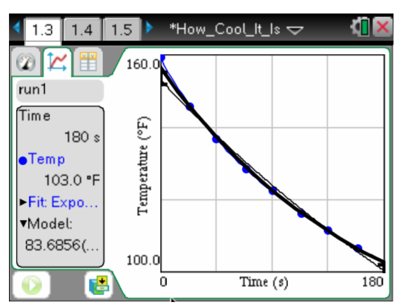


Use this Calculator page to find the regression equation for $y_{difference}$. Select **MENU > Statistics > Stat Calculations > Exponential Regression**. Choose run1.time for X List, difference for Y List, and Save RegEqn to: f1.

4. Write the regression equation for $y_{difference}$.

   Sample Answers: Answers will vary, depending upon the data that the students collect. One possible answer is: $y_{difference} = 83.6856\,(0.9943)^x$.

Move back to page 1.3.



Use the equation for $y_{difference}$ to create a better model to fit the data using the form $y = y_{difference} + y_{room\ temperature}$. Select **MENU > Analyze > Model**. Enter the information for $y_{difference} + y_{room\ temperature}$, and press enter.

5. Write the equation that you entered as your model.

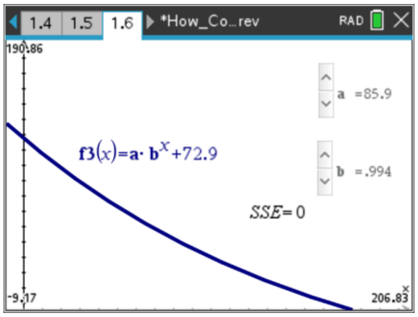
Sample Answers: Answers will vary, depending upon the data that the students generate. One possible answer is: $y = 83.6856\,(0.9943)^x + 72.9.$

6. Explain the similarities and differences between this equation and the original exponential regression equation.

Sample Answers: Both equations model exponential decay functions. The parameters, *a* and *b*, are slightly different. However, the biggest difference between the two equations is the horizontal asymptote. The original equation had a horizontal asymptote of y = 0. The new equation has a horizontal asymptote of y = room temperature.

Teacher Tip: You might want to have students create their own model of the function $y = a \cdot b^x + y_{room\ temperature}$.

Move to page 1.6.



The data from the original data set is shown and labeled as (run1.time, run1.temperature) and is modeled by the equation $f3(x) = a \cdot b^x + 72.9.$

The Sum of Squares due to Error, SSE, is also given on this page and can be used to measure how well the model fits the data. Smaller values indicate that the model fits the data better.

Click on the arrows of the sliders to change the values of the variables *a* and *b*. Try to find values for *a* and *b* such that the SSE is as small as possible.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

7. Write the equation that has the smallest SSE value. How does your equation compare to the regression equation that you entered as your model on Page 1.3?

Sample Answers: Students answers will vary. They might find that the equation that best fits the data is the one they used as their model for question #5.

8. Based on the equation that you created, what would the temperature be three minutes after the water begins to cool?

Sample Answers: Answers will vary, depending upon students' equations. If we use the equation $y = 83.6856\left(0.9943\right)^x + 72.9,$ and substitute *x* = 180 (3 minutes = 180 seconds), we see that the temperature would be approximately 102.8°.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 4 at the end of this lesson.

Tech Tip: You might want students to insert a Scratchpad or Calculator page to compute the answers for questions #8 and 9.

9. a. Based on the equation that you created, how long would it take for the temperature of the water to reach 78°F?

Sample Answers: If we solve the equation $83.6856\left(0.9943\right)^x + 72.9 = 78$ for x, we obtain x = 489.446 meaning that it takes 489.446 seconds, or approximately 8 minutes, for the water to cool down to 78°.

b. When would the temperature reach 32°F?

Sample Answers: Never, unless the room temperature is below 32°.

c. Explain your answers.

Sample Answers: To determine when the water temperature will reach $78,$ solve the equation $83.6856\left(0.9943\right)^x + 72.9 = 78$ for x, and we obtain x = 489.446.
To determine when the water temperature will reach $32,$ solve the equation $83.6856\left(0.9943\right)^x + 72.9 = 32.$ This equation has no solution.
Unless the temperature of the room is less than $32^\circ,$ the water will never reach $32^\circ.$ The horizontal asymptote for the function $y = 83.6856\left(0.9943\right)^x + 72.9$ is y = 72.9. We see that as *x* gets larger and larger, 0.9943^x gets smaller and smaller, and $83.6856\left(0.9943\right)^x$ also gets smaller and smaller. We can say that $\lim_{x\to\infty} 83.6856\left(0.9943\right)^x = 0.$ Thus,

$\lim_{x \to \infty} 83.6856\left(0.9943\right)^x + 72.9 = 72.9.$ This tells us that the temperature of the water will never go below 72.9 , and will therefore never reach 32° .

10. If the initial temperature of the hot water in a room with temperature 70° is $170^\circ,$ and it cooled 1% every second, write an equation that would model the water temperature as the water cools. What do the two parameters represent?

    Sample Answers: Since the initial water temperature is $170^\circ,$ and room temperature is $70^\circ,$ we set *a* $= 170 - 70 = 100^\circ.$ Because the water cools at a rate of 1% every second, b = 1 – 0.01 = 0.99. Thus, the equation can be written as $y = 100 \cdot 0.99^x + 70.$

11. Hot drinks such as coffee, tea, and hot chocolate seem to cool slowly when we have to wait to drink them and then cool rapidly once they reach the temperature at which we would like to drink them.
    a. Is it true that very hot drinks cool very slowly at first, and then cool rapidly after reaching a more reasonable temperature?

    Sample Answers: No, hot drinks do not cool slowly at first. People might feel this way when they first get a hot liquid because they are impatient. On the contrary, the most dramatic temperature change occurs at the start of the experiment, when temperature is highest.

b. Explain your answer through use of the cooling data, the graph, and/or the equations you generated for this activity.

Sample Answers: Since the rate of change is a percentage of the initial temperature, the water cools more rapidly at first. If we look at the graph, we see that the slope of the curve is steeper at first and then gets less steep as time goes by.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The importance of adjusting a regression equation to fit data in a real world setting.
- How to incorporate the SSE value in determining an equation to model data.
- How to use the model to interpolate and extrapolate information.

TI-Nspire Navigator

Note 1

Question 1, Name of Feature: Screen Capture and Live Presenter

You might want to leave Screen Capture running in the background, with a 30 second automatic refresh, and without student names displayed. This will enable you to monitor students' progress and make the necessary adjustments to your lesson.

You might want to ask one of the students to serve as the Live Presenter and demonstrate to the class how to navigate through the activity.

Note 2

Question 2, Name of Feature: Quick Poll

You might want to use a Quick Poll to have students send in their regression equations. You can then compare the similarities and differences in their equations.

Note 3

Question 6, Name of Feature: Screen Capture

You might want to use Screen Capture to compare the various SSE values that students obtained.

Note 4

Question 8, 9, and 10, Name of Feature: Quick Poll

You might want to use a *Quick Poll* to have students send in their answers to questions #8, 9, and 10.