Verbal to Visual: Sketching Graphs

Math Objectives

- Students will sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- Students will describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).
- Students will model mathematics.
- Students will attend to precision.

Vocabulary

- function
- increasing/decreasing function
- constant rate of change
- quadrants

About the Lesson

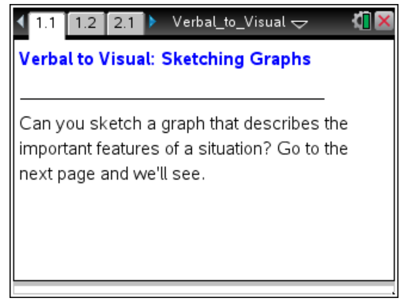
- This lesson involves sketching graphs based upon important features described verbally in a situation.
- As a result, students will:
  - Focus on identifying important features that aid in drawing sketches of functions, such as:
    - where the function is increasing, decreasing, or remaining constant;
    - the slope of the function or parts of the function, and;
    - the quadrants the function lies in.
  - Use these same ideas to describe the functional relationship between two variables as they write their own situation for a partner student to analyze.

TI-Nspire™ Navigator™ System

- Send out the *Verbal_to_Visual.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Verbal_to_Visual_Sketching_Graphs_Student.pdf
- Verbal_to_Visual_Sketching_Graphs_Student.doc

TI-Nspire document

- Verbal_to_Visual.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (👆) getting ready to grab the point. Press [ctrl] [click] to grab the point and close the hand (✊). Press [esc] to release the point or deselect other objects.

Tech Tip: If students experience difficulty grabbing and dragging a point, have them tap and hold the desired point for a few seconds and then drag the point to the desired location.

Tech Tip: Use [tab] to select the arrows that are minimized sliders. Then use the arrows on the handheld to easily turn on/off Draw, Erase draw, Show function, or reveal the next Problem. If the function entry line is shown, press [esc] to hide it again.

Below are four scenarios that provide clues about certain features of graphs. Work with a partner and make a sketch of a graph on the TI-Nspire™ that you believe the scenario describes.

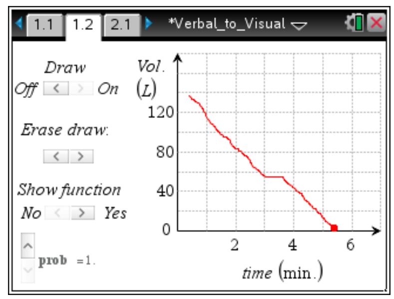
- To make a sketch, grab the red point on the screen and move the point to where you want to start your graph.
- Change the "Draw" to "On." When you move the red point, you will begin to draw your graph.
- If you want to start drawing again, you can select either arrow of "Erase Draw," change the "Draw" to "On," and then move the red point to start the new drawing.
- When you believe you have a good approximation for your sketch, use the "Show function" slider to see how your sketch matches another graph showing the important features of the function.
- Compare the important features between the two sketches. It is **not likely** that the sketches will be **exactly identical** but the **important features** should be very similar.

For example, if the scenario indicates the function should be decreasing at a constant rate across a certain period of time, then a negative slope is an important feature that should match. If no specific slope was provided in the problem, there could be many different negative values for a slope. Do not worry if the slopes don't match exactly just be sure both are negative. What is important is that you and your partner can determine the important features of the graphs that are provided in the scenarios and sketch them appropriately.

> **Teacher Tip:** You might need to emphasize to the students that they are making sketches and that only at key points do their graphs need to match exactly to the sample sketch provided. All of the screenshots provided in this file are 'sample'

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. **Problem 1**: You remove the plug from a bathtub holding about 150 liters of water, and it begins draining at a constant rate. After 3 minutes, you put the plug back in to remove the small bath toys so they don't go down the drain. That takes about 30 seconds, so the draining stopped for that length of time. You then remove the plug again and let the tub empty. That takes another 2 minutes. What might a graph of that situation look like?
   a. Underline important pieces of information from the scenario that you can use to draw your sketch.
   b. Draw your sketch on the TI-Nspire.
   c. Draw a final copy of your sketch below.

> **Teacher Tip:** Consider the level of your students, but to increase the rigor of this activity and satisfy additional standards for your curriculum, consider adding a part e for each problem. After step d, describing the graph verbally, have students do an algebraic step e to the right of the graph they drew where they write the piecewise function. For students who need some scaffolding to write a piecewise function for their scenario, write the following on the board
>
> $$V(t)=\begin{cases} \_\_\_t+\_\_\_ & ,\ 0\le t\le 3 \\ \_\_\_ & ,\ 3\le t\le 3.5 \\ \_\_\_(t-\_\_\_)+\_\_\_ & ,\ 3.5\le t\le 5.5 \end{cases}$$
>
> where the blanks for the first piece are the student's rate, or slope, and the y-intercept. The second piece is a horizontal line, so it is the volume for those 30 seconds. The third piece is set up for point-slope form.

Sample Answers: Students might underline the following:

You remove the plug from a bathtub holding about 150 liters of water and it begins draining at a constant rate. After 3 minutes, you put the plug back in to remove the small bath toys so they don't go down the drain. That takes about 30 seconds so the draining stopped for that length of time.

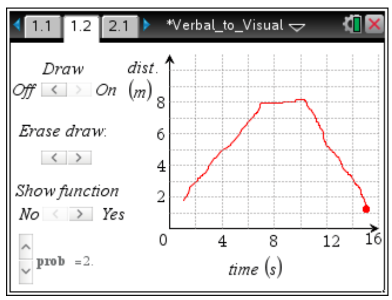
You then remove the plug again and <u>let the tub empty</u>. That takes <u>another 2 minutes.</u>

> **Teacher Tip:** When modeling with mathematics simplifications of the real-world situations are often made. The bathtub actually does not drain at a constant rate according to Torricelli's law (which can be derived from Bernoulli's principle). The weight of the water causes the tub to drain faster at first. As an optional extension have the students draw and discuss how the graph would look different. The rate of decrease of volume would be faster (steeper on graph) at first and then become slower (less steep on graph).

d. In your own words, describe how the important pieces of the scenario are shown in your sketch to support why you made the graph look the way it does.

<u>Answer:</u> We started our graph at 150 on the y-axis and drew our line going down since the bathtub is draining and the volume is decreasing. We drew a line because the tub is draining at a constant rate. When time was at 3 minutes we drew a horizontal line to show the 30 seconds of stopping up the bathtub. Then we continued drawing the line going down until we hit the x-axis at 5.5 minutes since the bathtub drained for another 2 minutes before it got empty. A volume of zero represents an empty tub.

2. **Problem 2:** You and a friend are using a motion detector (or range finder). The detector keeps track of your distance away from it over a period of 15 seconds. You hold the detector facing away from you to collect motion information from your friend. She stands 1 meter from the detector and starts to walk away from you at a constant rate for 7 seconds. She stops for 3 seconds, and then walks backwards towards you faster than she walked away and arrives where she started after 5 seconds. What might a graph of this situation look like?



a. Underline important pieces of information from the scenario that you can use to draw your sketch.
b. Draw your sketch on the TI-Nspire.
c. Draw a final copy of your sketch on the next page.

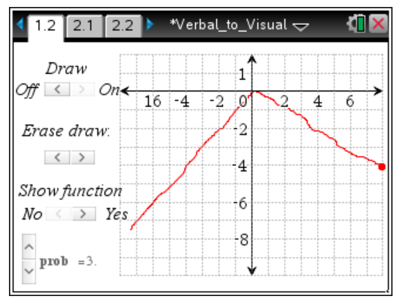
<u>Sample Answers:</u> Students might underline the following: You and a friend are using a motion detector (or range finder). The detector keeps track of your distance away from it <u>over a period of 15 seconds</u>. You hold the detector facing away from you to collect motion information from your friend. She <u>stands 1 meter from the detector</u> and starts <u>to walk away from you at a constant rate</u>

for 7 seconds. She stops for 3 seconds, and then walks backwards towards you faster than she walked away and arrives where she started after 5 seconds.

d. In your own words, describe how the important pieces of the scenario are shown in your sketch to support why you made the graph look the way it does.

Answer: We started the graph with a distance of 1 meter since the girl started there. Since she walks away, the distance is increasing so the line has an upward slope. It is a line because she is walking at a constant rate. After 7 seconds, she stops, so we drew a horizontal line to show how her distance is not changing from the motion detector. It was horizontal for a time of 3 seconds. We knew we had to be back to the beginning distance of 1 meter and we had 5 seconds to walk there. We also knew the slope had to be steeper than it was in the beginning since she was walking faster.

3. **Problem 3:** Suppose you have a function that is increasing at a constant rate in the 3rd quadrant until the origin. At the origin the function is now decreasing at a slower constant rate than it was increasing. What might a graph of that function look like?



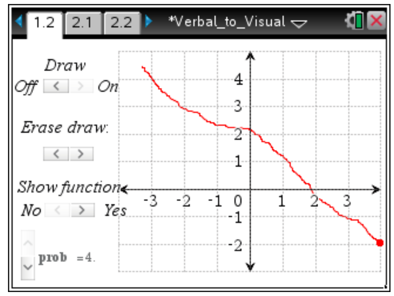
a. Underline important pieces of information from the scenario that you can use to draw your sketch.
b. Draw your sketch on the TI-Nspire.
c. Draw a final copy of your sketch below.

Sample Answers: Students might underline the following:
Suppose you have a function that is increasing at a constant rate in the 3rd quadrant until the origin. At the origin the function is now decreasing at a slower constant rate than it was increasing.

d. In your own words, describe how the important pieces of the scenario are shown in your sketch to support why you made the graph look the way it does.

Answer: We started in the 3rd quadrant and drew a straight line with an upward slope to represent the function increasing at a constant rate. At the origin (0,0), we stopped and then graphed a line going downward to represent a decreasing function. It is not as steep as our first slope since we knew the rate was decreasing slower than it was increasing.

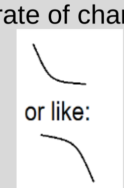
4. **Problem 4:** Suppose you have a function that is decreasing in the 2nd quadrant until the point (0,2) and it is not decreasing at a constant rate. At the point (0,2) it begins to decrease at a constant rate. What might a graph of this function look like?



a. Underline important pieces of information from the scenario that you can use to draw your sketch.
b. Draw your sketch on the TI-Nspire.
c. Draw a final copy of your sketch below.

Sample Answers: Students might underline the following:
Suppose you have a function that is decreasing in the 2nd quadrant until the point (0,2) and it is not decreasing at a constant rate. At the point (0,2) it begins to decrease at a constant rate.

Teacher Tip: The language "it is not decreasing at a constant rate" is left for the student to think about: what does "decreasing not at a constant rate" look like on a graph? They know a straight line suggests a constant rate of change so their function cannot look "straight". Functions that have curves would suggest a non-constant rate of change and could possibly look like:



and still be decreasing. You might want to have class captures of the graphs students make to help further the discussion on graphs that are decreasing at a non-constant rate of change and how they know it is a non-constant rate of change.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

d. In your own words, describe how the important pieces of the scenario are shown in your sketch to support why you made the graph look the way it does.

Answer: We started in the second quadrant and drew our graph going downward to represent the decreasing function until we crossed the y-axis at (0,2). Since it wasn't decreasing at a constant rate, we knew we couldn't draw a straight line, so we put a curve in the graph to show it decreasing but not at a constant rate.

Move to page 2.1.

5. a. Write your own scenario that you can share with your partner to sketch a new graph. Your scenario needs to involve two variables (for example: time and temperature, or cost of a ticket and number of people) and should provide enough important features of the graph so that your partner can make a reasonable sketch. Once you have finished your scenario, trade papers with your partner and draw new sketches.

Sample Answers: Student answers will vary.

b. Draw a sketch of your own graph below before you swap scenarios.

Sample Answers: Sketches will vary.

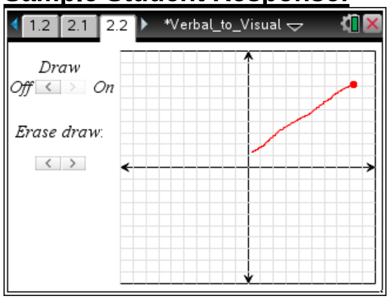
Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to identify important features from a verbal/written description that aid in drawing sketches of functions.
- How to make appropriate sketches of graphs of functions based upon important features described verbally in a situation.
- How to write their own scenario describing a function, providing important features for others to refer to when drawing a sketch of the function.

Suppose we are going to stretch a spring by hanging weights on it—the more weight you hang on the spring, the more it stretches. We'll start with a spring that is 1 decimeter in length when no weights are attached. We will progressively add eight 50-gram weights, one at a time. As the weights are added, the spring stretches at a constant rate until, with the last weight, it has stretched to a length of 6 decimeters. What might a graph of that situation look like?

Sample Student Response:



TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of this lesson.

TI-Nspire Navigator

Note 1

Question 1a, Class Capture

As students complete their first sketches, you might want to use Class Capture to see the variety of sketches and have the whole class determine, PRIOR to selecting *Show function* on the document page which show critical features of the graph described in the scenario.

Note 2

Question 1d, Class Capture

As students make their sketches, you might want to use Class Capture to see how students are making sense of a decreasing function that is not decreasing at a constant rate.

Note 3

Assessment, Class Capture

Use Class Capture to view all sketched graphs and allow students to discuss which graphs match the given scenario.