Solid of Revolution

Student Activity

7 8 9 10 11 **12**

TI-84CE | Activity | Student | 60 min

Introduction

A liquid flows into an irregular shaped vase at a constant rate, is it possible to describe the changing height of the fluid using mathematics? In this investigation you will collect data for the height of a liquid as it flows into a vase, produce mathematical equations to model these changes and rates, determine an equation for the volume of the liquid at any height *h*, and finally, produce a 3D model of the vase!

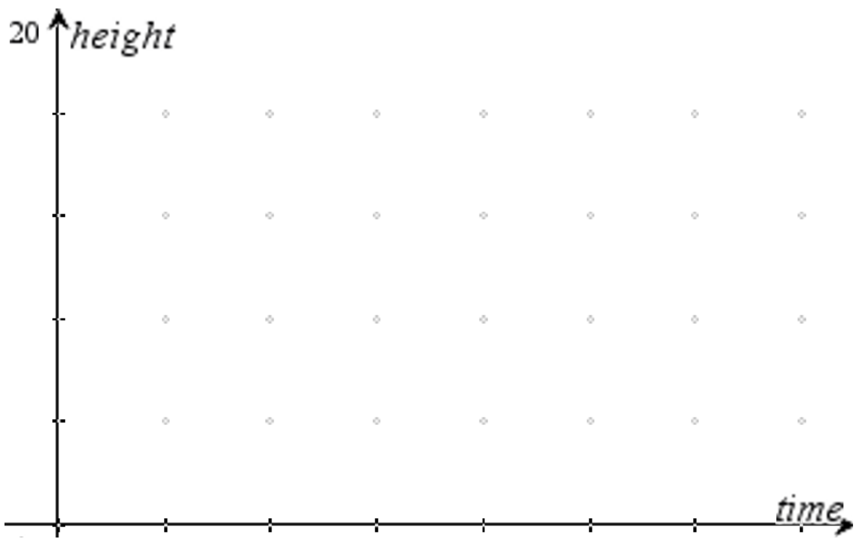
Equipment

- Image of Vase
- YouTube video
- TI-84CE Calculator
- TI-Connect CE software
- Calculator to Computer Link cable

Filling the Vase

Question: 1.

Imagine the vase pictured above is being filled at a constant rate to a depth of 20cm (as shown). If this process takes 90 seconds, sketch a graph of the height versus time.



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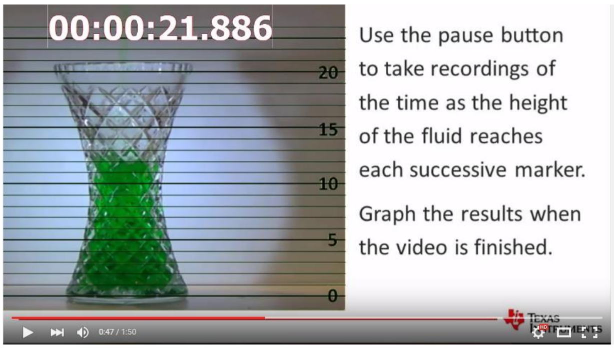
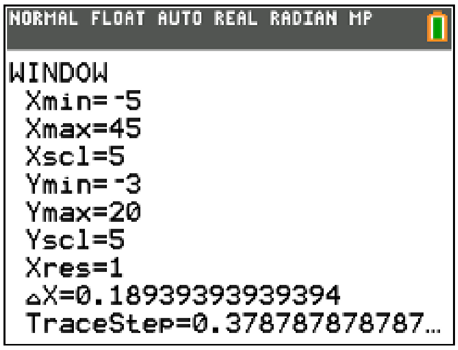
Use List 1 (L_1) to record the time and List 2 (L_2) for the height of the liquid as the vase in the video is filled at a constant rate[1]. Use the video speed and pause option on YouTube to slow and stop the video as the liquid reaches each graduation mark. Note that the splashing and crystal structure of the vase make some reading difficult; however they are a part of the real problem.



Related Rates and Solid of Revolution
http://bit.ly/JugFillingVase2

Graph your data as a scatter plot with TIME (L_1) on the independent axis and HEIGHT (L_2) on the dependent.

Approximate window settings are shown opposite.



Question: 2.

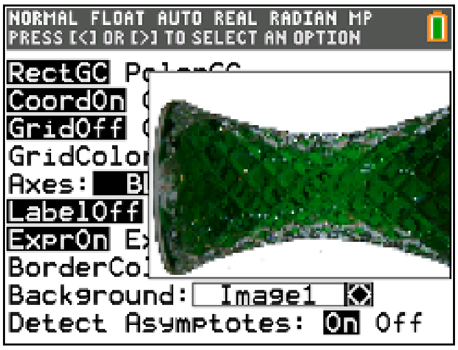
Relate key features of the graph to the shape of the vase including a comparison of your sketch graph with the one obtained from the video data.

Volume of the Vase

Note: Make sure your document settings have Float = 5 or 6. Due to the practical nature of the questions in this section, calculations need a high level of accuracy in order to model the shape of the vase.

Use the TI-Connect CE software[2] to drag and drop the image into your calculator. You can also use a calculator to calculator cable and the LINK option [2nd] [xtθn] on the calculator to send the image from one calculator to another.

Use the FORMAT menu to place the image of the vase in the background.



Two sets of window settings will be used repeatedly in this activity. To quickly recall previous window settings use [zoom] and move across to select Memory at the top of the screen. Current window settings can be saved using Zoom Sto and later recalled Zoom Rcl.

[1] Filled at a constant rate = The change in volume with respect to time is constant. This quantity can be computed using the amount of time it takes to fill the vase and knowing the volume of the vase: 800ml at 18cm depth.

[2] TI-84CE Connect Software (FREE) = MAC version: http://bit.ly/TI84CE_Link_MAC PC version: http://bit.ly/TI84CE_Link_PC

Turn off the Scatter Plot and adjust the window settings as follows:

XMin = -1 XMax = 20

YMin = -6.5 YMax = 6.5

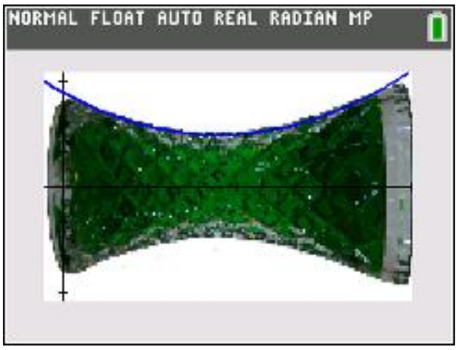
These window settings produce a true aspect ratio and scale as it relates to the image of the vase.

Use the [Trace] key to determine an approximate coordinate for the turning point of a parabola that will model the curvature of the vase.

Question: 3.

Determine the equation for a quadratic function that represents the curvature of the vase.

Important: Place your equation in Y_1



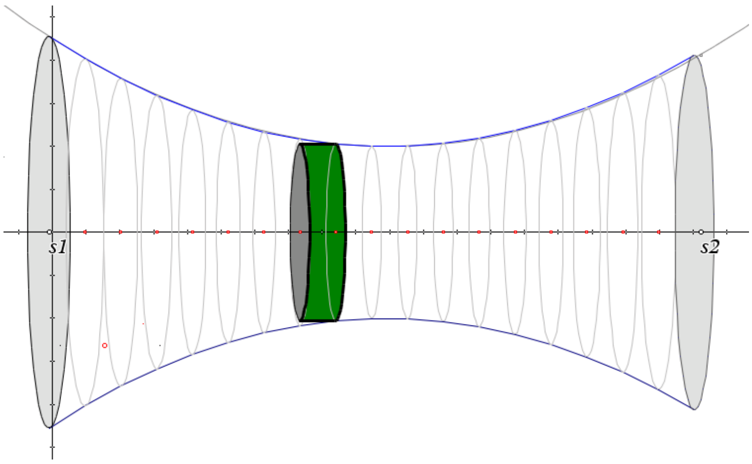
Question: 4.

Explain why the vase is easier to model when tipped on its side.

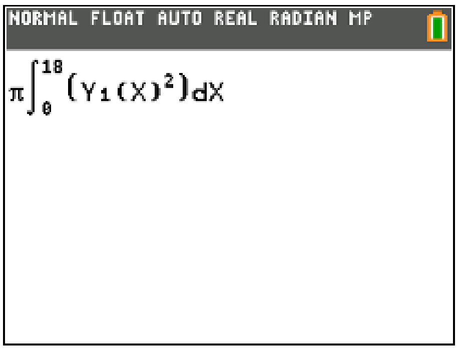
The theoretical volume of the vase can be determined by rotating the function around the *x*- axis.

$$V = \pi \int f(x)^2 dx$$

The actual volume of the vase is 800ml when filled to a depth of 18cm, the amount shown in the original picture.



The calculator can be used to determine the approximate volume of liquid in the vase using the syntax shown opposite.



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Question: 5.

According to the calculator, what is the volume of the vase when filled to a depth of 18cm? Discuss the accuracy of the result including the potential cause for any discrepancies.

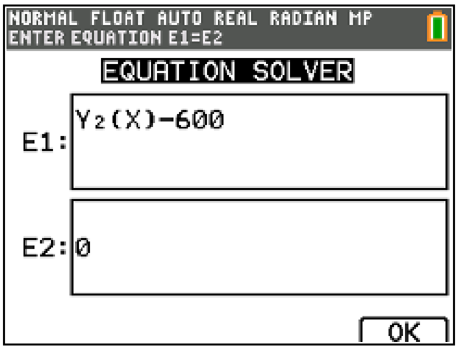
Question: 6.

Use calculus to determine an expression for the volume of liquid (v) in the vase as a function of height (*h*).

Store this function in Y_2.

Question: 7.

Use the Numeric Solver on the calculator to determine an approximate value for the height of the liquid in the container when it contains 600ml.
Note: Use the function stored in Y_2 to 'paste' the equation into the solver, as shown opposite.



Question: 8.

Given that the liquid is flowing into the vase at a constant rate, check your answer to question 7 using your data.

Graphing the Height

The aim of this section is to determine a rule that describes the ***rate*** at which the height of the liquid is changing using a combination of the formula generated for the volume (based on the shape of the vase) and compare this to the data that was collected from the video.

Consider the following:

$$\frac{dv}{dt} = constant$$

This can be determined from information provided and collected already.

$$v(t) = \int constant\ dt$$

Remember that the initial volume of fluid was 0 ml.

Question: 9.

Determine a rule for: $v(t)$

Question: 10.

Using the answer to Question 6 and Question 9, determine a rule relating *time* and *height*.
Note: This rule is best expressed as $t(h)$.

Question: 11.

Change the settings for the scatterplot to have *height* on the independent axis (x axis) and *time* on the dependent (y axis) and then graph the function from Question 10. Discuss how accurately the function matches the data.
Note: A variation of 0.01 in the original formula for the curvature of the vase makes a significant difference in the final relationship.

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