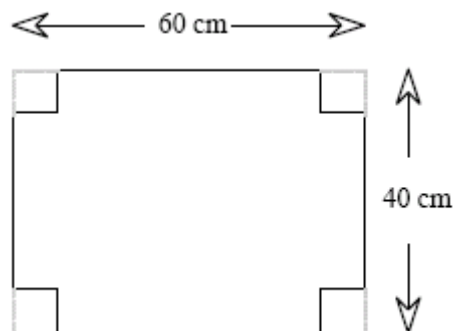




The Problem

The physical sciences teacher needs several open-topped boxes for storing laboratory materials. She has given the industrial technologies class several pieces of metal sheeting to make the boxes. Each of the metal pieces is a rectangle measuring 40 cm by 60 cm. The class plans to make the boxes by cutting equal-sized squares from each corner of a metal sheet, bending up the sides, and welding the edges. The goal is to get the box with the largest possible volume.



Explore the volume by making a table with columns for height and volume. Define the relationship in terms of two variables, x and y . Press $\boxed{Y=}$ and enter $(60 - 2x)(40 - 2x)x$ in **Y1**.

Now define the limiting values for the viewing window. To define the limits on x , consider the equation $Y1 = (60 - 2x)(40 - 2x)x$ that provides the computed volumes and think about how the equation is used to answer our original problem. Given the conditions from the problem situation, x must take on values between 0 and 20.

1. Why must x have a value between 0 and 20?

Press $\boxed{\text{WINDOW}}$ and enter the following values. When entering the negative values, be certain to use $\boxed{-}$ and *not* $\boxed{\ominus}$.

$$\begin{array}{lll} X_{\min} = -1 & X_{\max} = 21 & X_{\text{scl}} = 1 \\ Y_{\min} = -100 & Y_{\max} = 9000 & Y_{\text{scl}} = 500 \end{array}$$

Press $\boxed{\text{GRAPH}}$.

2. In your group, write a brief explanation of why the graph should look the way it does based on your previous work with the volume table and the patterns you observed there.

Press $\boxed{\text{TRACE}}$.

3. Do the coordinates of points on the graph agree with the values you computed earlier in this activity? Explain what these two values mean with respect to the box volume problem.

Use \blacktriangleleft and \blacktriangleright to view the coordinates of points on the graph. Move the cursor until you locate the point on the graph that gives the *largest y-coordinate*.

4. What are the values of x and y at the highest point on the graph?

$$x = \underline{\hspace{10em}} \quad y = \underline{\hspace{10em}}$$



Use the arrow keys to determine the x-coordinates of the points immediately to the left and to the right of the point found above.

5. Record the x-coordinates.

Point to left: _____

Point to right: _____

The solution you are looking for lies somewhere between these two x-values. Check to see that these tabular solutions fall somewhere between the two values you have written above.

*(Hint: Press **2nd** [TABLE] to access the table of numbers.)*

Press **ZOOM** and select **ZBox**. Use the arrow keys to move the cursor somewhere above and to the left of the highest point of the graph. Once satisfied with the cursor's location, press **ENTER**. Now move the cursor to the right and down until you reach the opposite corner of the box you are forming. When satisfied with the location, press **ENTER** again.

Use **TRACE** and the arrow keys again to locate where you believe the maximum value to be.

6. Record the coordinates of your estimated maximum value.

$x =$ _____ $y =$ _____

7. By observing the x-coordinates of the points on each side of this point, you can again determine an interval that contains the desired solution. What x interval contains the value associated with the maximum height of the graph?

If you zoom one more time on the place where you believe the maximum volume will be, you will be able to provide a value for x that is accurate to the nearest tenth.

Press **ZOOM** and select **Zbox** again. Make another box around the area on the graph where you think the maximum volume lies. Press **TRACE** to find new x-values for an interval containing the maximum volume as you have done previously.

8. Write the new interval for the maximum volume. Determine whether you can provide an x-value accurate to the nearest tenth. Remember, you want all numbers in your interval to round to the same tenth's value.

9. Based on the results from your trace of the graph, what size squares should the class cut from the pieces of sheet metal? What are the dimensions and volume of the box with the largest volume?

Side length of square: _____

Height: _____ Length: _____

Width: _____ Volume: _____

10. Discuss in your groups the advantages of investigating the box volume problem graphically. Write those advantages below.