Integration by Substitution

ID: 9890

Time required
45 minutes

Activity Overview

In this activity, we explore methods for computing integrals of functions not in one of the standard forms. The focus here is upon the use of substitution to transform the given integral into a standard form. The approach taken is largely symbolic and makes full use of the computer algebra facilities of TI-Nspire CAS. Prepared algebraic interactive Notes pages are utilized for skill development and consolidation.

Topic: Techniques of Integration

- *Use trigonometric substitutions to compute integrals involving rational functions.*
- *Use substitutions such as* $u = \sqrt{ax+b}$ *to compute an integral.*

Teacher Preparation and Notes

This investigation offers opportunities for review and consolidation of key concepts related to methods of substitution and integration of composite functions. Opportunities are provided for skill development and practice of the method of taking integrals of suitable functions. Care should be taken to provide ample time for all students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build new and deeper understanding.

- *This activity is intended to be* ***teacher led.*** *You should seat your students in pairs so they can work cooperatively on their handhelds. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. The majority of the ideas and concepts are only presented in this document, so be sure to cover all the material necessary for students' total comprehension.*
- *This activity can serve to consolidate earlier work on integration. It offers a suitable introduction to integration by substitution.*
- *Begin by reviewing the method of differentiation of composite functions (the "chain rule"), and methods of integration of the standard function forms.*
- *This activity requires the use of CAS technology.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution .tns files and student worksheet, go to education.ti.com/exchange and enter "9890" in the keyword search box.***

Associated Materials

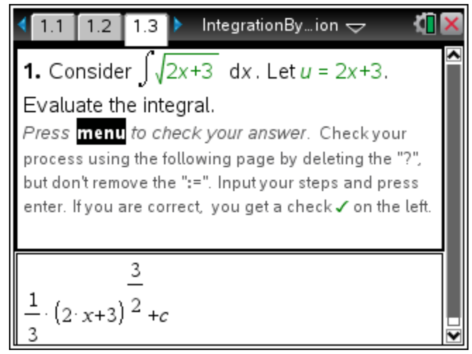
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *IntegrationBySubstitution_Student.doc*
- *IntegrationBySubstitution.tns*
- *IntegrationBySubstitution_Soln.tns*

Problem 1 – Introduction

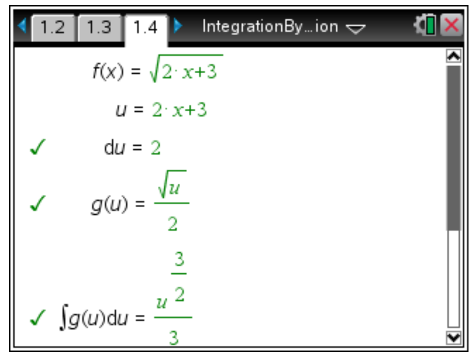
Begin with discussion and review of both the chain rule for differentiation of composite functions and of the integrals of standard function forms.

Ensure that students are comfortable with these and then challenge them to consider more difficult forms—in this case, composite functions of the form $y = f(g(x))$ which may be suitable for integration by substitution methods.



Students are scaffolded in their application of integration by substitution through the availability of an algebraic *Notes* page, set up for this purpose.

The function to be integrated is entered after f(x), then the choice of substitution, ***u***, after “ux:=”. Students enter the derivative, ***du/dx*** after du=, and then substitute ***u·du*** for ***f(x)·dx.*** They can then integrate with respect to ***u***, and finally replace to give the result in terms of ***x***. Each step is checked for algebraic equivalence.

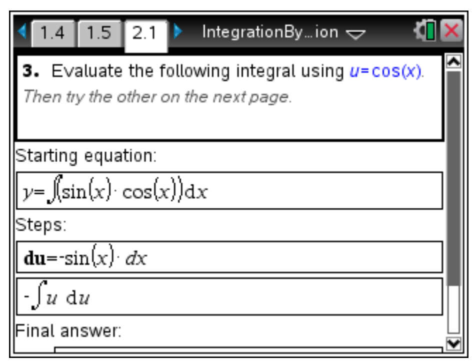


TI-Nspire™ Navigator™ Opportunity: *Quick Poll and Class Capture*

See Note 1 at the end of this lesson.

A series of carefully chosen questions follow, with the substitution specified for the first few and the spreadsheet available for support. The integral for **sin(*x*)·cos(*x*)** is developed in three different ways, and then other function types follow.

<u>**Note**</u>: $\sin(x)\cos(x)dx$ is transformed into $\frac{1}{2}\sin(2x)$ using the Double Angle formula.

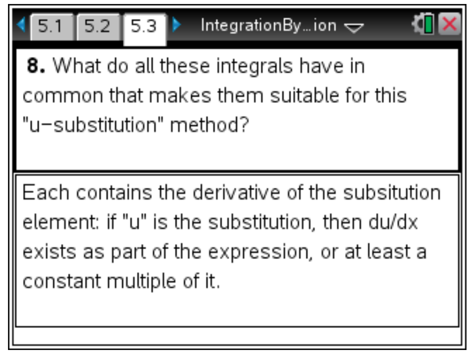


TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Problem 2 – Common Feature

After working through several different examples, students are challenged to identify the common feature: each of the given functions in some way includes the derivative of the function to be substituted.



This realization is critical for students to understand that this method will not work for all functions, but only for certain well-chosen forms.

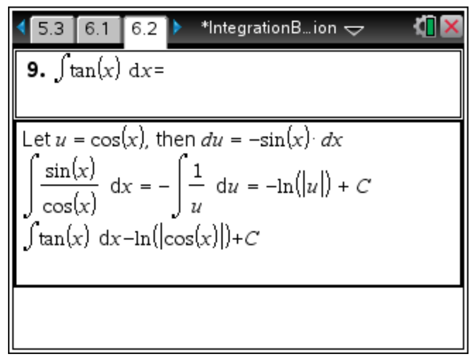
Note: Make sure that students remember that $\int \frac{1}{u} du = \ln|u| + C$. The spreadsheets will not accept ln(u) as a correct answer. Students must include absolute value.

Extension

The last two questions have been selected to offer a little more challenge and require some insight and rearrangement using their knowledge of trigonometric identities and relationships.

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cos(x)^3 = \cos(x)\cos(x)^2 = \cos(x)(1-\sin(x)^2)$$



Student Solutions

1. $u = 2x+3$; $du = 2dx$; $\int \sqrt{2x+3}\,dx = \frac{1}{2}\int u^{\frac{1}{2}} du = \frac{1}{3}u^{\frac{3}{2}} + C = \frac{1}{3}(2x+3)^{\frac{3}{2}} + C$

2. $u = \sin(x)$; $du = \cos(x)dx$; $\int \sin(x)\cos(x)dx = \int u\, du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin(x)^2 + C$

3. $u = \cos(x)$; $du = -\sin(x)dx$; $\int \sin(x)\cos(x)dx = -\int u\, du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\cos(x)^2 + C$

4. $u = 2x$; $du = 2dx$; $\int \frac{1}{2}\sin(2x)dx = \frac{1}{4}\int \sin(u)du = -\frac{1}{4}\cos(u) + C = -\frac{1}{4}\cos(2x) + C$

5. $u = x^2 + 2x + 3$; $du = (2x + 2)dx$;
$\int \frac{x+1}{x^2+2x+3}dx = \frac{1}{2}\int \frac{1}{u} du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x+3| + C$

6. $u = \cos(x)$; $du = -\sin(x)dx$; $\int \sin(x)\, e^{\cos(x)}dx = -\int e^u\, du = -e^u + C = -e^{\cos(x)} + C$

7. $u = 4x^2 + 1$; $du = 8x\, dx$; $\int \frac{x}{4x^2+1}dx = \frac{1}{8}\int \frac{1}{u} du = \frac{1}{8}\ln|u| + C = \frac{1}{8}\ln|4x^2+1| + C$

8. Each contains the derivative of the substitution element: if u is the substitution, then du/dx exists as part of the expression, or at least a constant multiple of it.

9. $u = \cos(x)$; $du = -\sin(x)$; $\int \frac{\sin(x)}{\cos(x)}dx = -\int \frac{1}{u}du = -\ln|u| + C = -\ln|\cos(x)| + C$

10. $u = \sin(x)$; $du = \cos(x)dx$;

$$\int \cos(x)(1-\sin(x)^2)dx = \int (1-u^2)\,du = u - \frac{u^3}{3} + C = \sin(x) - \frac{\sin(x)^3}{3} + C$$

TI-Nspire™ Navigator™ Opportunities

Note 1
Problem 1, *Quick Poll and Class Capture*
Page 1.3 can be used as a “Just in Time” Quick Poll to check for students’ preliminary understanding and provide the motivation to learn about a process which helps ensure they get the correct answer.

This would be a good place to do a Class Capture to verify that students are entering the correct values into the table and answering the questions correctly. They should delete the question mark “?”, but not the colon equals “:=”. They also need to press enter to evaluate the Math Box.

Note 2
Problem 1, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. Any page with a question space like page 2.1, where students can even show their steps, can be used as a Quick Poll.

