Building Rules

Introduction

There are several other similar activities that involve students building relationships from visual diagrams; some of these involve proof as a final step. The TI-Nspire files in many cases can be substituted as Power Point slide shows, the animations are generally very easy to make, however the benefit of the TI-Nspire file is that students control the pace at which the information is being provided.

One of the aims of this activity it to build a rule that describes the number of blocks required to complete a particular pattern. The pattern may be seen in many tiling problems, a rule to determine the total number of pavers or tile is therefore an efficient means of calculating requirements if the number of tiles is likely to be very high. The real focus of this problem however, is to explore different methods for deriving a rule. One of the skills for which mathematicians are valued is their ability to solve problems in an efficient manner. Having only a single strategy is unlikely to make for efficiency, so this activity explores three different ways to solve the same problem.

Problem 1 – The Algebraic Approach

Open the TI-nspire document "Building Rules" and navigate to page 1.2. The slider in the bottom right corner of the page can be used to change the pattern number (S). Start with pattern number one (S=1) and record the number of blocks (in total) required to build the pattern.

Since: 'S' has been used in the slider to record the pattern number, all rules should use an x instead.

The number of new blocks being added each time is shown in yellow to help highlight the difference between consecutive patterns.
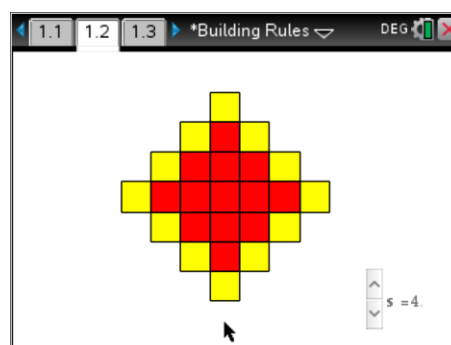
The [TAB] key can be used navigate quickly to the slider. To increase the slider amount while the slider is active (shown by black surrounding line) click up/down on the touchpad.

The fastest way to send students a copy of the file is via the TI-Navigator system. Use the 'send' option to send the document to the entire class wirelessly.

If the Navigator system is not available, the Teacher Software contains a "Transfer Tool". This provides continuous processing so that as each student connects to the teacher's computer they receive the file without any further actions required by the teacher.

The final option is have students send the file from handheld to handheld. Providing a class challenge and timing them means that the files generally get around the room in a couple of minutes!

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Question: 1.

Record the pattern number (x) and the number of blocks (bl) required to produce the entire pattern.

Pattern (x)	Blocks (bl)
1	1
2	5
3	13
4	25
5	41

Enter the results into the spreadsheet on Page 1.3. The 'difference' columns (δ_1 and δ_2) automatically determine the successive differences between blocks (δ_1) and the differences of these differences (δ_2). These are referred to as first and second order differences.

With reference to difference tables, when a constant difference is determined this means two things:

- The original relationship can be expressed as a polynomial
- The degree of the polynomial is equal to the 'difference order' where the first *constant* difference occurs.

Question: 2.

After entering the 'block' results, what do the values in δ_2 say about the relationship between the pattern number (x) and the number of blocks (bl)?

The differences in δ_2 are constant, therefore the relationship is 'quadratic', a degree 2 polynomial.

Question: 3.

Based on the results from Question 2, set up an appropriate general function that describes the relationship between the pattern number (x) and the number of blocks (bl). Using a selection of points from your data, use the solve command to determine the actual equation.

Students define: $bl(x) = ax^2 + bx + c$ and $solve(bl(1) = 1 \ and \ bl(2) = 5 \ and \ bl(3) = 13, \{a, b, c\})$

to get the results: $a = 2 \ and \ b = -2 \ and \ c = 1$ ... therefore $bl(x) = 2x^2 - 2x + 1$

Note: Students need to assign each parameter or redefine the function in order to use it.

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Question: 4.

Use a scatter plot in the graph application to display the pattern vs blocks data then graph the function defined in Question 3. Looking at the graph (and data), what domain restrictions should be applied to the function?

The graph matches the data points perfectly; this can be checked using the trace function or the calculator application. Problems with the graph include:
- The graph is continuous and the data and context of the problem are discrete
- The function is only valid for $x > 0$
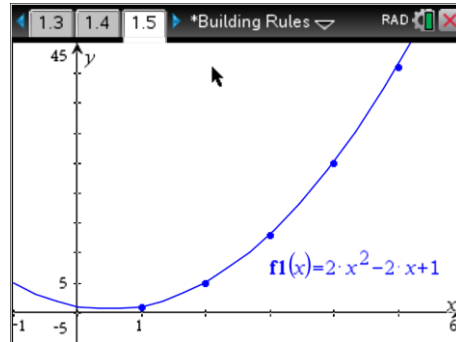
Problem 2 – CAS = Black Box

Imagine you are building the pattern by hand. The literal rule for building is: "add a square to each exposed side". The recently added squares are shown each time in yellow.
For the first stage (s = 1) no blocks are 'added', the pattern simply starts with a single block. To reach the next stage four blocks are added (s = 2).
The total number of blocks contained in the pattern is therefore the progressive **sum** of those added plus the single block required to start the pattern.
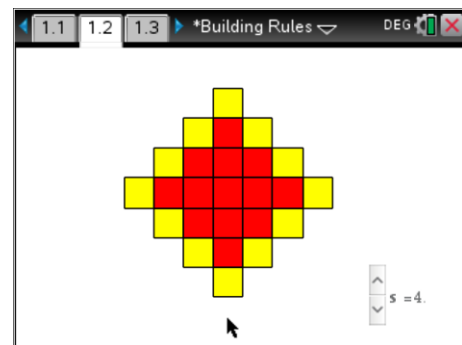
The progressive sum is generally denoted by the Greek symbol sigma: Σ. The sum template can be found in the calculus menu in the calculator application and also in the symbols palette.

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Question: 5.

Record the pattern number and the number of **yellow** blocks required to build each stage. The first amount has been entered. (Refer to the problem description for justification of the zero)

Pattern (x)	Blocks (bl)
1	0
2	4
3	8
4	12
5	16

Question: 6.

Write a rule $ad(x)$ relating the pattern number (x) and the number of blocks **added** each time.

The rule is easier as the function is linear. $ad(x) = 4(x-1) \ \ or \ \ ad(x) = 4x - 4$

Question: 7.

The rule for the total number of blocks required is the progressive sum of the blocks being added and can therefore be determined using:

$$1 + \sum_{x=1}^{n} ad(x) \ \text{--- where } n \text{ represents the final shape number.}$$

Write down the rule produced by the calculator and explain why the "1 +" is necessary.

"The summation rule only considers the blocks 'added' to the pattern, so the initial value needs to be added. "

This is very similar to an integration problem requiring " + c " and using initial conditions to resolve the general equation in order to find a specific solution.

Rule produced: $1 + \sum_{x=1}^{n} ad(x) = 2n^2 - 2n + 1$

Problem 3 – Visualising Algebra

Drawing a diagram is a very powerful problem solving tool. In this case the diagram has been drawn already, however to help focus thinking on specific aspects of the diagram a 'hint' button has been included.

With the hint off, scroll through the pattern as before.

Switch the hint on; then scroll through the pattern once again.

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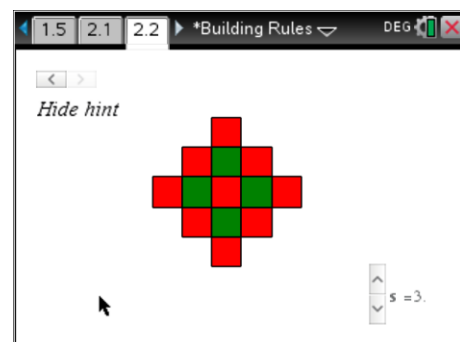
Question: 8.

Record the number of green and red blocks while the 'hint' is on. (Record the quantities only, no need to reference the colour). Use the pattern number and the quantities of red and green blocks to write a rule for the total number of blocks. Explain how you determined your rule.

The red / green combination occurs as:

Pattern Number	Red & Green Blocks	Total
1	$0 + 1 = 1^2$	1
2	$1 + 4 = 1^2 + 2^2$	5
3	$4 + 9 = 2^2 + 3^2$	13
4	$9 + 16 = 3^2 + 4^2$	25
5	$16 + 25 = 4^2 + 5^2$	41

From the table a simple conclusion for the total would be: $x^2 + (x-1)^2 = 2x^2 - 2x + 1$

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