What are the chances?

Teacher Notes and Answers

TI-Nspire   Investigation   Student   90 min

Introduction

In probability, we often look at likelihood of events that are influenced by chance. Consider the event of rolling a pair of six-sided dice and observing the sum of the two numbers that turn up. We can observe what might happen if we roll the dice a fixed number of times (the experimental probability), and then we can use mathematics to look at what we might expect to happen in the long run (the theoretical probability).

Part 1: Simulating chance events

Most calculators also have a random integer command, which generates random integers between two limiting values. For example, the command randInt(1,20) on the TI-Nspire CAS will generate a random integer between 1 and 20 (inclusive).

To do this on the TI-Nspire CAS

- Press **HOME-1** to create a new document, and then press **1** to add a **Calculator** page.
- Press **MENU-5** to select **Probability**, then press **4** (Random) and then **2** (Integer).
- Type **randint(1,20)** and then press · to display a random integer between 1 and 20.
- Press **ENTER** a few times to generate some extra random numbers.
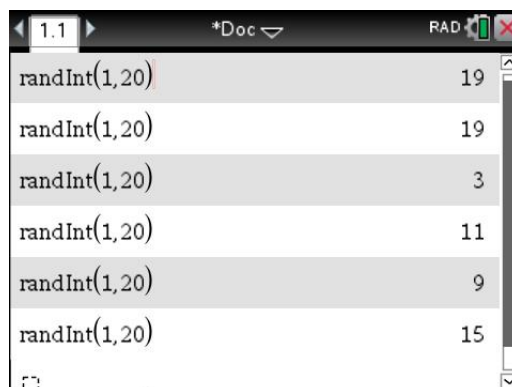
A number of random integers will be generated.

To begin with, we will simulate the process of rolling of a single die.

Question 1.
What command would simulate this experiment? randInt(1, 6)

Question 2.
Now try the command 60 times, and record the results in a frequency table (as shown below).

Die score	1	2	3	4	5	6
Tally						
Frequency						

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Question 3.

Observe your table results. Which score occurred most frequently?

Answers will vary, but the frequency of each die score will be close to 10. Given that only 60 trials are done, there may still be significant variation in the frequencies.

Question 4.

Compare your results with some other students near you. What do you notice?

Answers will vary, but the frequency of each die score will be close to 10. Given that only 60 trials are done, there may still be significant variation in the frequencies.

To simulate the rolling of two dice, and adding the resulting numbers, we can add two random integers.

Question 5.

What command would simulate the sum of the two numbers when two dice are rolled?

randInt(1, 6) + randInt(1, 6)

Question 6.

Try this command a number of times on your calculator.

a) What is the smallest value you can obtain as a sum of the two random integers? 2

b) What is the largest value you can obtain as a sum of the two random integers? 12

Question 7.

Now try the command 100 times, and record the results in a frequency table (as shown below).

Die sum											
Tally											
Frequency											

Question 8.

Comment on the distribution of results you obtained here.

The die sum frequencies will vary, but the middle sums (e.g. 6, 7 and 8) should occur most frequently.

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Part 2: Analysing results graphically

Generating a table of results for an experiment such as this would be time consuming if we were to conduct a large number of trials, and so we will use the calculator to help us store, visualise and analyse the results using statistical features. To begin with, note that the command **randInt(1,6,600) → diescore** will generate 600 integers between 1 and 6, and store the results in a statistical variable named diescore. This can be used to generate and store the results of a large number of trials for further analysis.

To do this on the TI-Nspire CAS

- Press **MENU-5** to select **Probability** and then press **4** (Random) and then **2** (Integer).
- Type the following to complete the command **randInt(1,6,600) → diescore**

(Note that the key for storing to a variable is **CTRL-VAR**.) The results of the 600 trials are displayed as a bracketed list as shown right.
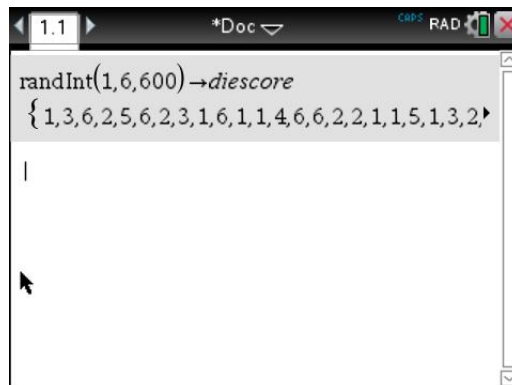
Question 9.

How many times would you expect the number 1 to occur in these 600 trials? Explain your reasoning.

We would expect a score of 1 to occur about 100 trials out of 600 (in the long term).

It is difficult to scroll through this list of 600 numbers to count the frequency of each number. However, the calculator has a counting feature that can provide this information. For instance, the command **countif(diescore,1)** will report the frequency of the number 1 in the 600 stored trials. [Note: The **countif** command can be found by pressing the CATALOG key – or just type it directly).]

Question 10.

Use this command to complete a frequency table for the 600 trials.

Die score	1	2	3	4	5	6
Tally						
Frequency						

Question 11.

Compare these frequencies with your answer to question 9. Are these results what you expected? Explain your answer.

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We can obtain a relative frequency of each score by dividing the frequency by the total number of trials.

Question 12.

Use this method to calculate the relative frequency of obtaining a die score of 1. Use the formula that follows to assist.

$$\text{Relative frequency of scoring 1} = \frac{\text{Frequency of scoring 1}}{\text{Number of trials}}$$

The experimental frequency of scoring a 1 will be close to, but not necessarily exactly 100, so the relative frequency will be close to 1/6 or 0.1666 ...

Question 13.

Copy and complete this frequency table for your previous results.

Die score	1	2	3	4	5	6
Tally						
Frequency						
Relative frequency (fraction)						
Relative frequency (decimal approx.)						

Question 14.

What would you expect the relative frequency of a die score of 1 to get close to if you conducted this trial a very large number of times? Express your answer as a fraction.
The relative frequency will be close to 1/6 or 0.1666 ...

The results of the experiment can be viewed graphically. To do this, we will create a histogram of the results.

On the TI-Npire CAS

- Press **MENU-5** to add a **Data and Statistics** page.
- Click at the bottom of the screen to add the variable **diescore** to the plotting screen. (This creates a dot plot of the results of the 600 trials.)
- Press **MENU-1** to select **Plot Type** and select **3** to select **Histogram**.

The results of the 600 trials are displayed as a histogram.
[*Note that the columns are for numbers in the range 1–<2, 2–<3, and so on, rather than centred on the numbers 1 to 6.*

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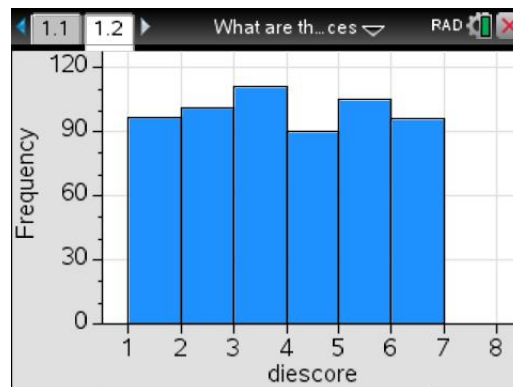
Question 15.

How would you expect this histogram to change in appearance if you were displaying the results of a much larger number of trials?

It would be expected that the histogram columns become closer in height; that is, their frequencies will converge to same number as the number of trials increase. This is because each score on the die is equally likely.

Now we will use this method to analyse the sum of two dice.

Question 16.

What command using the **randInt** function will simulate the results of 1000 rolls of two dice, and store the results in a statistical variable named **diesum**?

randint(1,6,1000) + randint(1,6,1000) → diesum

Press **CTRL-LEFT ARROW** to re-display the **Calculator** page, and enter this command.

Question 17.

Use the **countif** command to calculate the frequency of a sum of 2 in the 1000 stored trials.

Answers will vary, but the expected number would be 1/36 x 1000, or about 28.

Question 18.

Copy this table and use this command to complete the table for the 1000 trials.

Die sum	2	3	4	5	6	7	8	9	10	11	12
Tally											
Frequency											
Relative frequency (fraction)											
Relative frequency (decimal approx.)											

Question 19.

Are these relative frequencies what you expected? Explain your answer.

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Question 20.

Use your CAS to display a histogram of these results. Draw a sketch of this histogram and comment on its shape. Click on each column to verify the frequencies listed in the table above.

Sample screen shown below
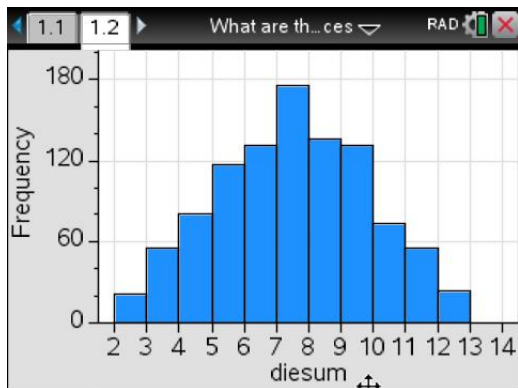
Part 3: Long-term relative frequency

To work out what we expect to happen in the two-dice sum experiment in the long run, we will look at all the possible outcomes when two dice are rolled.

Question 21.

Copy and complete the following table to display all the possible outcomes when rolling two dice.

		Die 1 score					
		1	*2*	*3*	*4*	*5*	*6*
	1	1,1	2,1	3,1	4,1	5,1	6,1
	2	1,2	2,2	3,2	4,2	5,2	6,2
Die 2 score	*3*	1,3	2,3	3,3	4,3	5,3	6,3
	4	1,4	2,4	3,4	4,4	5,4	6,4
	5	1,5	2,5	3,5	4,5	5,5	6,5
	6	1,6	2,6	3,6	4,6	5,6	6,6

Question 22.

For this experiment, how many possible outcomes are there in total? 36

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The probability of a particular sum can be calculated as follows.

$$\text{Probability of a sum of 3} = \frac{\text{Number of outcomes with a sum of 3}}{\text{Total possible outcomes}}$$

$$= \frac{2}{36}$$

$$\approx 0.056$$

Question 23.

Use the above method to calculate the probability of obtaining a sum of 2, 3, 4 ... 12, and then complete the table below.

Sum	2	3	4	5	6	7	8	9	10	11	12
No. of ways to obtain this sum	1	2	3	4	5	6	5	4	3	2	1
Probability (fraction)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
Probability (decimal)	0.028	0.056	0.083	0.111	0.139	0.167	0.139	0.111	0.083	0.056	0.028

Question 24.

Compare these theoretical probabilities with the relative frequencies obtained in your answer to question 17 and 18, and comment.

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Teacher notes

- This task is designed to familiarise students with random number generators and their use in simulating chance events. It is suitable for students in Years 9–11.

- The first part introduces the basic random integer function, and how it can be modified to generated random numbers within a specified range. Then students use a similar command to generate the appropriate random integer from 1 to 6 inclusive.

- Most software, including the TI-Nspire CAS, use pseudo-random numbers, which sample from a number sequence known to produce consecutive values with digits that have no apparent order. For this reason, it is sometimes necessary to 'seed' the random number generator, which permits the user to start from a different position in the sequence (e.g. RandSeed 98763524).

- Part 1 focuses on individual trials of the rolling of a die. This permits students to see the variation possible, and to appreciate the benefits of technology for reducing the tedium of trialling (as shown in part 2)!

- Part 2 focuses on generating results for a larger number of trials, and the use of count commands and histograms as tools for analysing these results. It also introduces relative frequency as an approximation to the theoretical probability of a particular outcome.

- Part 3 introduces the theoretical probability via the sum of two-dice experiment. Students compare the theoretical values with the relative frequencies obtained via the simulation.

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