Math Objectives

- Students will explore the relationship between chords of a circle and their perpendicular bisectors.
- Students will discover that the perpendicular bisector of a chord always passes through the center of a circle.
- Students will discover that a line perpendicular to a chord that passes through the center of a circle always bisects the chord. Students then apply these relationships to right triangles.
- Students will discover that two chords are congruent if and only if they are equidistant from the center of a circle.

Vocabulary

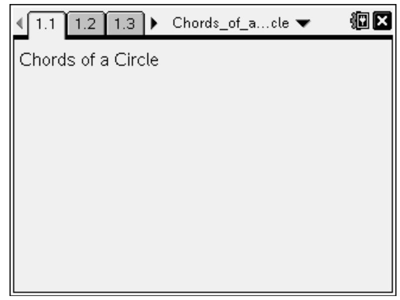
- chord of a circle
- perpendicular bisector
- equidistant

About the Lesson

- The estimated time for this activity is 45 minutes.
- For information regarding the creation of the file *Chords_of_a_Circle.tns*, refer to *Chords_of_a_Circle_Create.doc*.
- Students can either create the .tns file by following the instructions given in *Chords_of_a_Circle_Create.doc*, or they can use the pre-constructed file entitled *Chords_of_a_Circle.tns*.

TI-Nspire™ Navigator™ System

- Use Screen Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he/she used a certain tool.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing [ctrl] [G]. The entry line can also be expanded or collapsed by clicking the chevron.

Lesson Materials:

Create Instructions
Chords_of_a_Circle_Create.pdf

Student Activity
Chords_of_a_Circle_Student.pdf
Chords_of_a_Circle_Student.doc

TI-Nspire document
Chords_of_a_Circle.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (👆). Press [ctrl] [hand] to grab the point and close the hand (✊).

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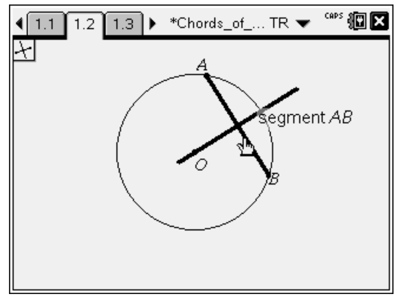

$\overline{AB}$ is a chord of circle *O* because its endpoints lie on the circle. Construct the perpendicular bisector of $\overline{AB}$ by pressing **Menu > Construction > Perpendicular Bisector**. Click $\overline{AB}$ and press [esc] to exit.

1. Drag point *A* or *B*. What do you observe about the perpendicular bisector of $\overline{AB}$?

<u>Answer:</u> The perpendicular bisector of $\overline{AB}$ always passes through point *O*, the center of the circle.

Teacher Tip: The points on a perpendicular bisector are those that are equidistant from the endpoints of the segment. Therefore the center of the circle is on that bisector.

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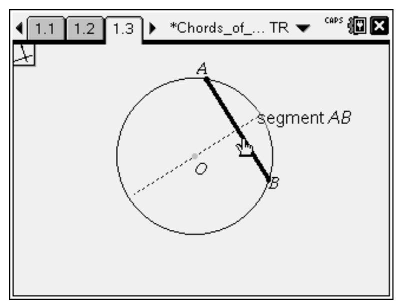
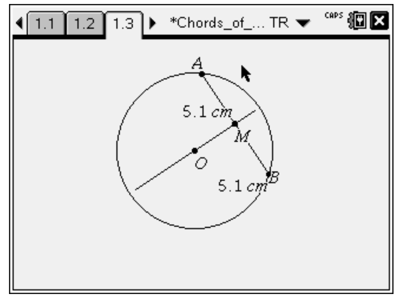


Construct a line through point *O* that is perpendicular to $\overline{AB}$. Press **Menu > Construction > Perpendicular.** Click point *O*, and then click $\overline{AB}$.

- Plot the intersection point of $\overline{AB}$ and the perpendicular line. Press **Menu > Points & Lines > Intersection Point(s)**. Click $\overline{AB}$ and the line perpendicular to $\overline{AB}$. Label this point *M* by immediately pressing [⇧shift] [M]. Press [esc] to exit.

- Measure the lengths of $\overline{AM}$ and $\overline{MB}$. Press **Menu > Measurement > Length**. Click point *A*, click point *M*, move the measurement to the inside of the circle close to the middle of $\overline{AM}$, and press [click]. Then click point *M*, click point *B*, move the measurement to the inside of the circle close to the middle of $\overline{MB}$, and press [click]. Then press [esc] to exit.



2. Drag point *A* or *B*. What is the relationship between $\overline{AM}$ and $\overline{MB}$?

<u>Answer:</u> The lengths of $\overline{AM}$ and $\overline{MB}$ are always the same. $\overline{AM} \cong \overline{MB}$.

> **Teacher Tip:** Why? Note that this is only a perpendicular construction (not a perpendicular bisector construction). You can use HL congruence between ΔAOM and ΔBOM. These are right triangles by perpendicular construction; *AO* and *BO* are radii and *OM* is a common side.

3. When the length of $\overline{AB}$ is as short as possible, what do you observe about $\overline{OM}$?

<u>Answer:</u> When the length of $\overline{AB}$ is as short as possible, points *A* and *B* converge to a single point and $\overline{OM}$ becomes the radius of the circle.

4. When the length of $\overline{AB}$ is as long as possible, what do you observe?

<u>Answer:</u> When the length of $\overline{AB}$ is as long as possible, $\overline{AB}$ passes through point *O* and becomes the diameter of the circle. Midpoint *M* coincides with center *O*.

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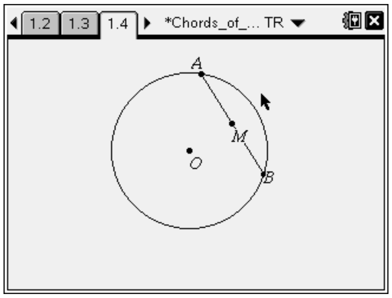

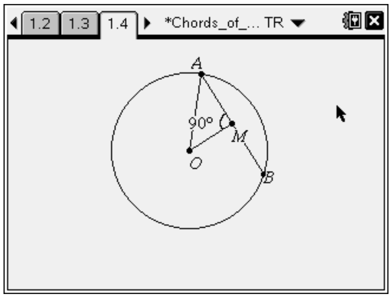

Construct the midpoint of $\overline{AB}$. Press **Menu > Construction > Midpoint**. Click $\overline{AB}$ and label this point *M* by immediately pressing [⇧shift] [M]. Press [esc] to exit.

- Create $\overline{OM}$ by pressing **Menu > Points & Lines > Segment**. Click point *O*, and then click point *M*. Press [esc] to exit.
- Measure Ð AMO by pressing **Menu > Measurement > Angle**. Click point *A*, then click point *M*, and then click point *O*. Press [esc] to exit. **Note:** you may need to grab and move either the letter *M* or the 90°.
  - Create radius $\overline{AO}$ by pressing **Menu > Points & Lines > Segment**. Click point *A*, and then click point *O*. Press [esc] to exit.

5. What type of triangle is ΔAMO?

   Answer: ΔAMO is a right triangle.

6. When given the lengths of any 2 sides of ΔAMO, what equation can be used to find the length of the third side?

   Answer: For a right triangle with leg length of *a* and *b* and hypotenuse *c*, the Pythagorean Theorem tells us $a^2 + b^2 = c^2$.

7. If $AB = 6$ and $AO = 5$, find the length of $\overline{OM}$.

   Answer: $\overline{OM} = 4$

Teacher Tip: Remind students to half the measure of segment AB.

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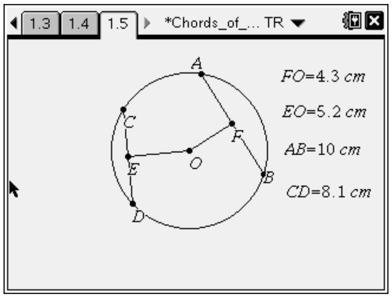

Drag points *A*, *B*, and *C* until $\overline{FO} \cong \overline{EO}$.

8. What is the relationship between $\overline{AB}$ and $\overline{CD}$?

Answer: The lengths of $\overline{AB}$ and $\overline{CD}$ are equal. $\overline{AB} \cong \overline{CD}$.

> **Teacher Tip:** Have a discussion with students about rounding. To avoid cluttering the screen, the document is showing only a few digits. It may be possible to have $FO = EO$ but $AB \neq CD$.

9. Drag points *A*, *B*, *C*, and *D* until $\overline{AB} \cong \overline{CD}$. What is the relationship between $\overline{FO}$ and $\overline{EO}$?

Answer: The lengths of $\overline{FO}$ and $\overline{EO}$ are equal. $\overline{FO} \cong \overline{EO}$.

> **Teacher Tip:** Same comment as above. It is possible to have $AB = CD$ but $FO \neq EO$.