

Fly Straight

Student Activity

7 8 9 10 11 12



Problem Statement

There are all sorts of patterns created by sequences such as the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13 ...

In this coding exercise we look at a very usual sequence with an aim to see if there are any observable patterns appear as the sequence progresses. The sequence starts with the first two terms defined: $t_0 = 1$ and $t_1 = 1$.

Term Number:	n	0	1	2
Term:	t_n	1	1	

Just like the Fibonacci sequence, this one is also recursive, meaning successive terms depend on previous terms. To calculate the next term in the sequence we check to see if there are any common factors between the highlighted values:

Term Number:	n	0	1	2
Term:	t_n	1	1	

If the highest common factor is 1 then we add the two highlighted terms then and add 1 to the result: $1 + 2 + 1 = 4$.

Term Number:	n	0	1	2	3
Term:	t_n	1	1	4	

We're ready to calculate the next term. We apply the same criteria, if the highest common factor between the two highlighted terms is 1 then we add the two highlighted terms and then add 1 to the result: $4 + 3 + 1 = 8$.

Term Number:	n	0	1	2	3	4
Term:	t_n	1	1	4	8	

We're ready to calculate the next term. Applying the same criteria we see that the highest common factor between the highlighted terms (8 and 4) is 4. When the highest common factor is not equal to 1, the term (8) is divided by the highest common factor, in this case 4. Our new term becomes: $8 \div 4 = 2$.

We'll do two more terms for practice.

Term Number:	n	0	1	2	3	4	5
Term:	t_n	1	1	4	8	2	

The highest common factor of the two highlighted terms is 1, therefore the next term is $2 + 5 + 1 = 8$.

Term Number:	n	0	1	2	3	4	5	6
Term:	t_n	1	1	4	8	2	8	

For the second time we see that the highest common factor of the two highlighted terms is not equal to 1. So the next term is equal to $8 \div 2 = 4$ (Highest common factor of 8 and 6 is 2)

Check the next two terms in the table below.

Term Number:	n	0	1	2	3	4	5	6	7	8
Term:	t_n	1	1	4	8	2	8	4	12	3

Question: 1.

Complete the next series of terms in the table. (Check the answers with your teacher before continuing)

Term Number:	n	8	9	10	11	12	13	14	15	16
Term:	t_n	3								

Question: 2.

Create a scatterplot for the data collected so far with the term number (n) on the x axis (independent variable) and the term on the y axis (dependent variable). Include a copy of your scatterplot in your answers and identify any patterns if they exist.

Coding the Sequence – Introduction

Start a new document and insert a Program.

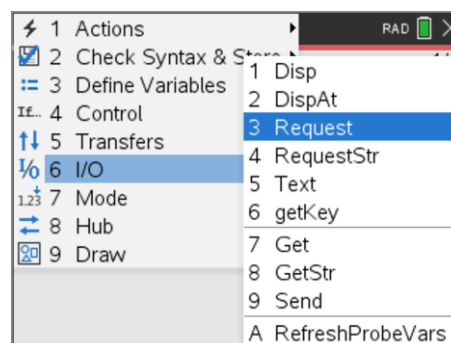
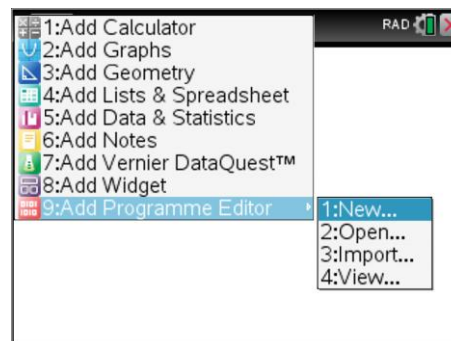
Name the program: Fly

Note: You cannot have spaces or special characters in a program name.

The first step is to prompt the user for the quantity of terms to be generated by the program.

Menu > I/O > Request

Request("How many terms? ", d)



The term numbers can be generated by a single command:

$$s:=Seq(x,x,0,d)$$

Enter the next line:

$$data:=\{1, 1\}$$

As the sequence is generated it will be stored in a list called data.

Save and run your program (Ctrl + R) and enter the quantity of terms when prompted. (Enter a value between 1 and 100)

The program will say 'done', but what has it done?

```

1.1 1.2 *Doc RAD 4/4
* fly
Define fly()=
Prgm
Request "How many terms?" ,d
s:=seq(x,x,0,d)
data:={1,1}
f:
EndPrgm

```

Question: 3.

Once you have run the program, in the calculator application type: s. What does s equal and represent?

Question: 4.

In the calculator application try the following calculations:

GCD(20,30) GCD(12,16) GCD(17,12) GCD(24,12)

Explain what the GCD command does.

Coding the Sequence – Making Decisions

It's time to start creating the sequence. The user has provided information about the quantity of terms to be generated. (d)

The first two terms are already stored in data. That means a LOOP can start at the third term and finish with term ' d '.

Menu > Control > For ... EndFor

The loop starts at 3 and ends at d . The loop variable is ' n '. (See opposite)

```

1.1 1.2 *Doc RAD 5/6
* fly
Define fly()=
Prgm
Request "How many terms?" ,d
s:=seq(x,x,0,d)
data:={1,1}
For n,3,d
EndFor
EndPrgm

```

Now for the decision process. The decision is based on whether or not the term number and the previous term have any factors in common.

Menu > Control > IF ... THEN ... ELSE ... ENDIF

Now the condition needs to be entered.

Condition: $GCD(data[n-1],s[n]) = 1$

Note:

The square brackets are used to refer to a specific element in a list. So $data[3] = t_3$ in mathematical terms.

Question: 5.

Explain what the condition $GCD(data[n-1],s[n]) = 1$ is testing.

If the condition has been met the program will follow the instruction immediately following THEN

$$\text{data}[n] := \text{data}[n-1] + s[n] + 1$$

```

1.1 1.2 *Doc RAD 6/10
* fly
Define fly()=
Prgm
Request "How many terms?" ,d
s:=seq(x,x,0,d)
data:={ 1,1 }
For n,3,d
If gcd(data[n-1],s[n])=1 Then
  data[n]:=data[n-1]+s[n]+1
Else

```

Question: 6.

Explain what this command does in reference to the sequence.

If the condition has not been met the program will follow the instruction immediately following ELSE

$$\text{data}[n] := \text{data}[n-1] \div \text{gcd}(\text{data}[n-1], s[n])$$

```

1.1 1.2 *Doc RAD 9/10
fly
data:={ 1,1 }
For n,3,d
If gcd(data[n-1],s[n])=1 Then
  data[n]:=data[n-1]+s[n]+1
Else
  data[n]:=data[n-1]/gcd(data[n-1],s[n])
EndIf
EndFor
EndPrgm

```

Question: 7.

Explain what this command does in reference to the sequence.

Mathematical Investigation - Flying Straight

The program is now ready to run.

Pressing Ctrl + R from the programming application.

Start by entering '20' for the number of terms when prompted. Once the program says "Done", recall the Data variable to ensure the terms are correct.

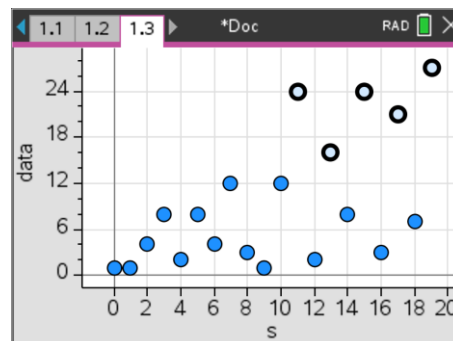
```

1.1 1.2 *Doc RAD
fly()
How many terms? 20
Done
data
{ 1,1,4,8,2,8,4,12,3,1,12,24,2,16,8,24,3,21,7 }

```

Insert a Data and Statistics application and plot the term number (s) on the x axis and the term (data) on the y axis for the first 20 terms in the sequence.

The aim of this investigation is to find patterns. Five points in the first 20 terms of the sequence have been highlighted (opposite). These points sit higher than most of the other points. Given that our sequence is based on divisibility (High Common Factor), will all of these points be prime numbers?



One of the challenges faced in any mathematical investigation is that we are not sure where the investigation may lead. We can only start with clues in our initial problem construction and then start generating data. Our first clue in this

investigation is that our sequence is based on divisibility. When divisibility exists (common factors), the value of the next term in the sequence decreases, the reverse is true when divisibility is not present.

It's now time to start investigating. The program provides a tool to quickly generate lots of terms, start small. Look for clues such as:

- Terms or term numbers with a particularly high quantity of factors such as 36, 60 and 120.
- Terms or term numbers that are prime numbers.
- Relative abundance of prime and composite numbers present in the sequence. (data)
- Progressively increase the quantity of data that you are analysing. Sometimes patterns start to appear when we get to stand back from the data.