



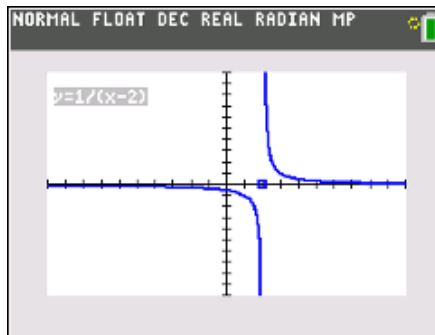
# Rational Functions

## Student Activity

Name \_\_\_\_\_

Class \_\_\_\_\_

In this activity, you will explore the properties of functions of the forms  $f(x) = \frac{1}{x-a}$  and  $f(x) = \frac{ax+b}{cx+d}$  using a program called **RATIONAL** that allows you to dynamically examine these rational functions.



Using the handheld, students will discover, or re-discover, the connection between a rational function, transformations, and both vertical and horizontal asymptotes. To accomplish this, they will be running a program call **RATIONAL**.

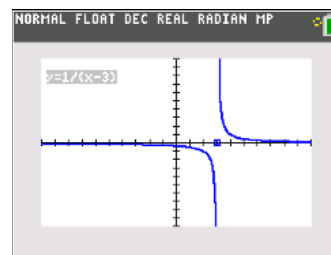
Let's review the idea behind vertical and horizontal asymptotes. A **vertical asymptote** is a vertical line that guides the graph of the function but is not part of it. It can never be crossed by the graph because it occurs at the x-value that is not in the domain of the function. A function may have more than one vertical asymptote. A **horizontal asymptote** is a horizontal line that the graph of a function approaches as the magnitude of the input increases without bound in either a positive or negative direction. A function may cross a horizontal asymptote for finite values of the input.

### Problem 1 – Using the Program RATIONAL to Discover Vertical Asymptotes

After the program has been downloaded onto the handheld, and from the home screen, press **prgm**, **1: TI-Basic**, and select the program **RATIONAL**. Press **enter** to run the program.



The program shows the graph of  $y = \frac{1}{x-a}$ . The value of  $a$  is shown by a dot on the x-axis at  $(a, 0)$ . The starting value of  $a$  is 3. Press the left and right arrows to adjust the value of  $a$ . Pressing the up or down arrows will exit the program. Notice that as the value of  $a$  changes, the equation and graph are updated.





**Discussion Questions**

1. With a classmate, press the left and right arrows and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the gap between the two curves.

2. Find the value of  $x$  where the function  $y = \frac{1}{x-2}$  is undefined.

3. Find the value of  $x$  where the function  $y = \frac{1}{x+1}$  is undefined.

4. Find the value of  $x$  where the function  $y = \frac{1}{x-a}$  is undefined.

5. Find the value of  $x$  where the function  $y = \frac{1}{x-a}$  has a vertical asymptote. With a classmate, come up with a rule to find the vertical asymptote.

**Extension Question:**

6. Discuss with a classmate what you notice about what happens horizontally as you move the slider left and right. Write an explanation and share with the class.

**Problem 2 – Using Graphs on the Handheld to Make Connections to Horizontal Asymptotes**



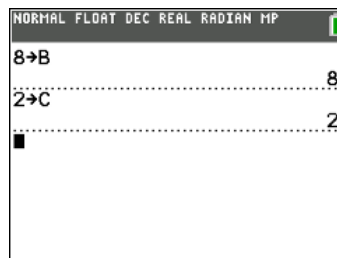
# Rational Functions

## Student Activity

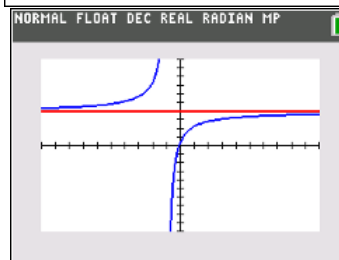
Name \_\_\_\_\_

Class \_\_\_\_\_

On your home screen, store 8 in for the letter B by pressing **8, sto, enter**. Then store 2 in for the letter C by pressing **2, sto, enter**. Go to the **y =** screen and enter the fraction  $\frac{b \cdot x + 1}{c \cdot x + 2}$  into **Y<sub>1</sub>** and press **enter**.



To make the connection once again to asymptotes, we will add a line. Enter the fraction  $\frac{b}{c}$  into **Y<sub>2</sub>** and press **graph**.



### Discussion Questions

1. With a classmate, look at the graph and discuss what you see. Write an explanation and share with the class what you notice about the relationships between the rational function and the horizontal line.

2. Find the value of  $y$  that the function seems to approach as the input values approach both positive and negative infinity when  $Y_1 = \frac{bx+1}{cx+2}$ .

3. Go back to the **y =** screen and change the equation in **Y<sub>1</sub>** to be  $\frac{cx+1}{bx+2}$  and the fraction in **Y<sub>2</sub>** to be  $\frac{c}{b}$ . Look at the graph. Find the value of  $y$  that the function seems to approach as the input values approach both positive and negative infinity when  $Y_1 = \frac{cx+1}{bx+2}$ .

4. Find the values of  $y$  for each function entered into **Y<sub>1</sub>** from questions 2 and 3 where you think each has a horizontal asymptote. With a classmate, come up with a rule to find the horizontal asymptote.



# Rational Functions

## Student Activity

Name \_\_\_\_\_

Class \_\_\_\_\_

---

### Further IB Application

The function  $f$  is defined by  $f(x) = \frac{8x-8}{3x-6}$  for  $x \in \mathbb{R}, x \neq 2$ .

- (a) Find the zero of  $f(x)$ .
- (b) For the graph of  $y = f(x)$ , write down the equation of
- (i) the Vertical Asymptote;
  - (ii) the Horizontal Asymptote.
- (c) (i) Find  $f^{-1}(x)$ , the inverse function of  $f(x)$ .
- (ii) Hence, or otherwise, find the value of  $f^{-1}(0)$ .