Trigonometric Ratios

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will explore sine, cosine, and tangent with respect to the ratios of the sides of right triangles.
- Students will use the TI-Nspire CX II to discover the relationship of these three trig functions.
- Students will solve any right triangle given an angle and the lengths of the opposite side, adjacent side or the hypotenuse.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- SOHCAHTOA
- sine, cosine, tangent
- adjacent
- opposite

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 3 Geometry and Trigonometry:

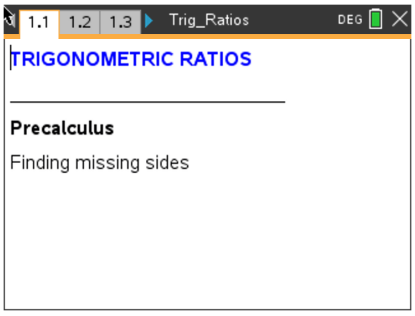
  3.2a Use of sin, cos, and tan ratios to find the sides and angles of right angled triangles

  3.3a Applications of right-angled trig
  - As a result, students will:

    Apply this information to real world situations

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

Trig_Ratios_Student-Nspire.pdf
Trig_Ratios_Student_Nspire.doc
Trig_Ratios.tns

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Problem 1 – Exploring trigonometric ratios

In right triangles, there is a relationship between the ratios of the side lengths and the trigonometric functions.

1. Using the triangle on **page 1.4**, find the following ratios and trig values to three decimal place.

   Answer: Remind students that the indicated value for the angle is rounded and not exact.

Ratio	Value
$\frac{a}{c}$	**0.750**
$\frac{b}{a}$	**0.889**
$\frac{b}{c}$	**0.667**

Trig Function	Value
sin(B)	**0.664**
cos(B)	**0.747**
tan(B)	**0.889**

2. Move to **page 1.5**. Based upon your answers, match each ratio with its correct trigonometric operation. **Answer:**

$\frac{b}{a}$ $\quad \sin(B) = \frac{b}{c}$

$\frac{b}{c}$ $\quad \tan(B) = \frac{b}{a}$

$\frac{a}{c}$ $\quad \cos(B) = \frac{a}{c}$

TI-Nspire Navigator Opportunity: *Live Presenter* See Note 1 at the end of this lesson.

3. Move to **page 2.1**.Test your hypothesis to see if your chosen relationships holds true. To do this, drag point *A* of each triangle on **pages 2.2**, **3.1** and **4.1**. Pick one value after you have dragged point *A* for each triangle and record the values in the table below.

Answer: Answers will vary. See a sample of the answer in the table. Point out to the students that as point A is being dragged around and the values of a, b, c, and B are changing, the calculated side length ratio and trig value is not.

Ratio	Value
$\frac{b}{c}$	$\frac{5}{8.6} \approx 0.581$
$\frac{a}{c}$	$\frac{4}{6.4} \approx 0.625$
$\frac{b}{a}$	$\frac{4}{7} \approx 0.571$

Trig Function	Value
$\sin(B) = \sin(35.54°)$	0.581
$\cos(B) = \cos(51.34°)$	0.625
$\tan(B) = \tan(29.74°)$	0.571

Based upon your answers hypothesize which ratio goes with each trigonometric function.

4. $\sin B$ = _____ ; $\cos B$ = _____ ; $\tan B$ = _____

Answer:

$\sin B = \frac{b}{c}$; $\cos B = \frac{a}{c}$; $\tan B = \frac{b}{a}$

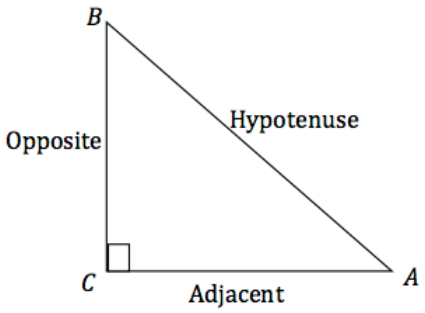
Teacher note: Make sure that students set the calculator's mode to degrees. To do this, move the cursor to the top right corner of the screen and if it says RAD, click on it with the touch pad and it will automatically change to degrees (DEG).

Move to **page 5.1**. A good acronym to use to help remember these relationships is SOHCAHTOA.

$$\sin A = \frac{Opposite}{Hypotenuse}$$

$$\cos A = \frac{adjacent}{hypotenuse}$$

$$\tan A = \frac{opposite}{adjacent}$$



Problem 2 – Trigonometry, what is it good for?

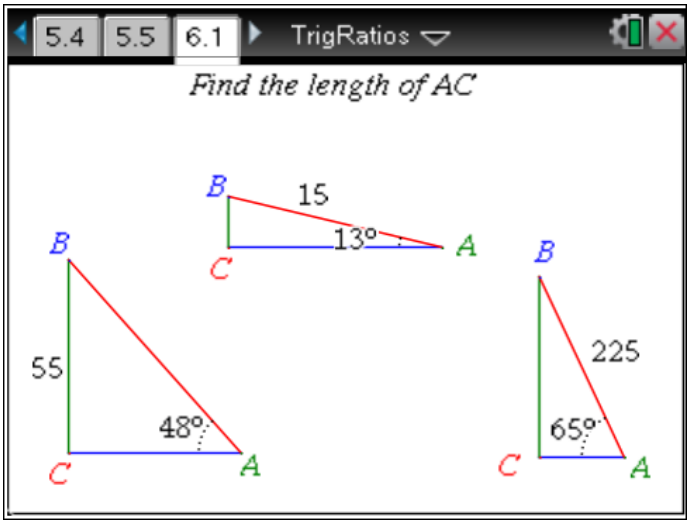
5. Move to **page 5.2**. One of the uses of trigonometry is finding missing side lengths of a triangle. On **pages 5.3-5.5**, use either sine, cosine, or tangent to find the length of the missing side. In the table below, write down the trig function relationship and then find the length of the missing side. Then, verify your answer by measuring the side.

Answer:

Page	Trig Function Relationship	Length of Missing Side
5.3	$\sin 24.7° = \frac{a}{27.5}$	$a = 27.5 \sin 24.7° \approx 11.5$
5.4	$\cos 47.7° = \frac{b}{18.6}$	$b = 18.6 \cos 47.7° \approx 12.5$
5.5	$\tan 41° = \frac{13.75}{b}$	$b = \frac{13.75}{\tan 41°} \approx 15.8$

TI-Nspire Navigator Opportunity: *Live Presenter* See Note 2 at the end of this lesson.

6. Move to **page 6.1**. Find the length of AC in each of three triangles. Record your answers on the screenshot below.



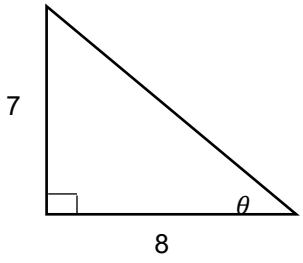
Answer: From left to right: $AC \approx 49.5$, $AC \approx 14.6$, $AC \approx 95.1$

TI-Nspire Navigator Opportunity: *Live Presenter* See Note 3 at the end of this lesson.

Further IB Extension

Another use of trigonometry is to use the ratio of sides of a right triangle to find the acute angles of a right triangle. In the following problem, you will not only find missing sides, but also missing angles of a right triangle.
First, let's do an example of finding a missing angle of a right triangle given its sides.

Find angle θ:



Since the sides given with respect to angle θ are the opposite side and the adjacent side, you will need to use tangent.

$$\tan\theta = \frac{7}{8}$$

How do we find the missing angle θ, given the sides? We will use The inverse tangent function $(tan^{-1}\ or\ arctan)$.

$$tan^{-1}\left(\frac{7}{8}\right) = 41.2°$$

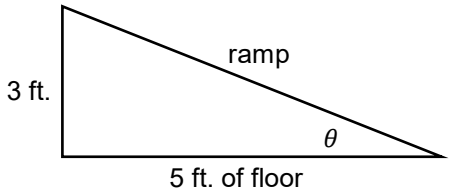
Problem

Suzie has realized that there is a problem with her dog. She loves to sleep in the bed with her, but she is too small to jump up on the bed or jump down off the bed. Being handy, she decides to construct a ramp that will allow her dog to easily get on and off the bed. Suzie realizes that she needs to do a little trigonometry to make this work. Unfortunately her bedroom is not very large so she does not have unlimited space for the ramp. She measures the height of the bed to be 3 feet high and that there is 5 feet of floor space for the ramp.

Using the trigonometric relationships discussed earlier in the activity, in parts (a) and (b) find:

(a) The angle of the ramp created with the floor, also known as the angle of elevation.

Answer:



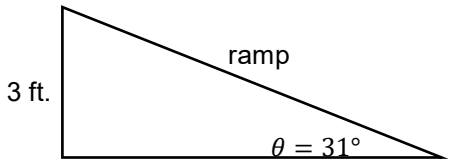
$\tan\theta = \frac{3}{5}$

$\theta = tan^{-1}\left(\frac{3}{5}\right)$

$\theta \approx 31.0°$

(b) The length of the ramp.

Answer:



$\sin 31° = \frac{3}{ramp}$ or $\cos 31° = \frac{5}{ramp}$

$ramp = \frac{3}{\sin 31°}$ $ramp = \frac{5}{\cos 31°}$

$ramp \approx 5.82\ ft.$ $ramp \approx 5.83\ ft.$

(c) With a classmate, discuss other ways to find the length in part (b) and what other considerations you must think of for the ramp.

Possible Answer: Students can find the length of the ramp using sin or cos, but also the Pythagorean Theorem. Some considerations students should discuss is the weight of the dog, materials to use, the ramp surface material, is the ramp permanent or could is be movable, etc.

TI-Nspire Navigator Opportunities

Note 1

Question 2, *Screen Capture*
Do a class *Screen Capture* on page 1.5 to check that students correctly drag each trig operation to its proper ratio.

Note 2

Question 5, *Screen Capture*
Consider using *Screen Capture* to monitor student progress as they work through pages 5.3 to 5.5.

Note 3

Question 6, *Quick Poll*
Consider sending a *Quick Poll* for student answers to the three problems on page 6.1.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*