Action Research Report # 6

TI-Nspire™ CAS learning handheld helps pre-service math teachers in Algebra and Calculus.

Teacher/Researcher:	Douglas A. Lapp
Location:	Central Michigan University, Mount Pleasant, MI
Course:	Problem-based Algebra & Calculus for Secondary Teachers
Grade:	Teacher education
Student Profile:	10 Fall Semester, 17 Spring Semster, undergraduates
Technology:	TI-Nspire CAS learning technology

Setting:

Central Michigan University (CMU) is situated in Mount Pleasant, a small college town on the Chippewa River in Michigan. With more than 20,000 on-campus students, it is the fourth largest university in the Michigan system. The CMU math teacher training program emphasizes rigor in mathematics as well as pedagogy.

Curriculum & Teaching:

Dr. Douglas A. Lapp is developing a textbook for this course as part of an NSF grant. The course meets twice during the week for 75 minutes throughout the full semester. The course models problem-based teaching technique with students working in subgroups. Student activities that contribute to their grade include reports and quizzes on readings, field observations, weekly quizzes using the TI-Nspire CAS handheld, a midterm test on content, assignments and labs, pedagogical reflections and a final project. During the course of the semester, Dr. Lapp estimates that 90% of the planned curriculum is covered.

Dr. Lapp uses the TI-Nspire CAS handheld along with other calculators and a projector. As the semester progresses, Dr. Lapp reports that both student activities and his own questioning evolve. Early in the semester, students tend to use TI-Nspire technology daily to check and compare answers, and weekly for the higher-level activities of generating new concepts, making predictions and discussing problem-solving strategy. As the course progresses, these higher-level activities become a daily feature of work in the class. Dr. Lapp reports that his own questioning changes over the course of the semester. Early on, there is daily attention to procedures of operating the TI-Nspire CAS handheld and math procedures. Later, there is less time on these instructions and skills, allowing more time for constant student focus on eliciting reasons and predictions for problems and conclusions. Throughout the course, students use multiple representations of problems that involve equations, graphs and tables. They also create their own TI-Nspire documents on a daily basis. About once a week, they use documents created by Dr. Lapp.

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Dr. Lapp reports that he felt quite comfortable with TI-Nspire CAS technology after only one week of use, especially since he had been involved in the technology's development. He feels that his students became proficient in using all (multiple) representation modes after approximately a month of use.

Results:

Dr. Lapp reports that the ability to dynamically connect representations is the biggest advantage of TI-Nspire technology. He gives this example of how he posed a problem to his students and they used multiple representations to solve the problem, building their own deeper understanding of the behavior of functions.

> After reflecting on posed secondary school curriculum materials and a classroom vignette, the pre-service teachers are asked the following:
>
> > In the example given above, a specific function, $f(x)=\sqrt{x}$, and a specific starting point, $x=2$, were used to generate the sequence. In this exploration, consider a more general question. Suppose you have a function *f* and the infinite sequence, $f(a)$, $f(f(a))$, $f(f(f(a))),\ldots$. Explore this sequence. What questions might you ask?

The pre-service teachers discuss the question in groups and then the instructor acts as a facilitator for the whole class to flush out their questions and assign different groups to explore different aspects of the problem as proposed by the students. Whether the students were exploring the effects of the initial seed value, the family of functions, the parameters of a specific family of functions or the characteristics of various families of functions, all of the groups took advantage of the dynamically-linked representations to see patterns and make conjectures.

I will now describe a general approach used by the students. Most groups began with a numeric representation by defining a function and then generating a spreadsheet that recalled the function over and over, showing values for the sequence, $f(a)$, $f(f(a))$, $f(f(f(a))),\ldots$ (See Figure 1).

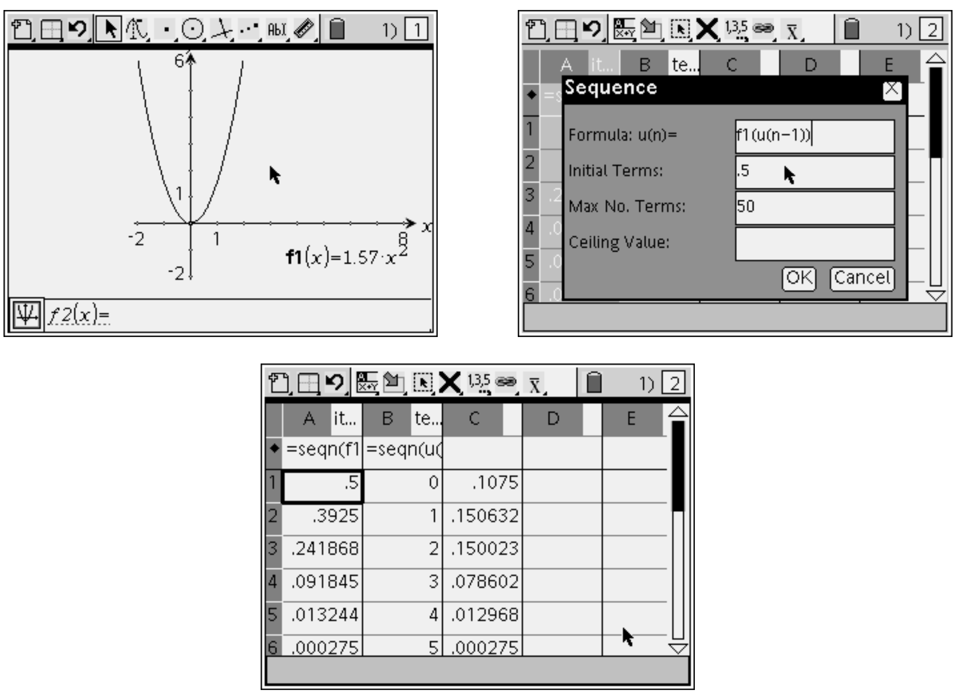


Figure 1: Generating the Sequence $f(a)$, $f(f(a))$, $f(f(f(a))),\ldots$

To gain some insight into the behavior of the sequence, the students graphed a scatter plot of the sequence placing it in a window side-by-side with the graph of the function (See Figure 2). In doing so, they expected the sequence to converge since the seed value being used was $x_0 = 0.5$ and they knew that raising a number, $|x_0| < 1$, to an exponent greater than 1 would result in convergence to zero. The scatter plot confirmed their conjecture; however, the group decided to investigate various forms of the function $f(x) = ax^2$. It was this decision that generated cognitive conflict for the students. In other words, the students expected convergence no matter what value was used for *a*.

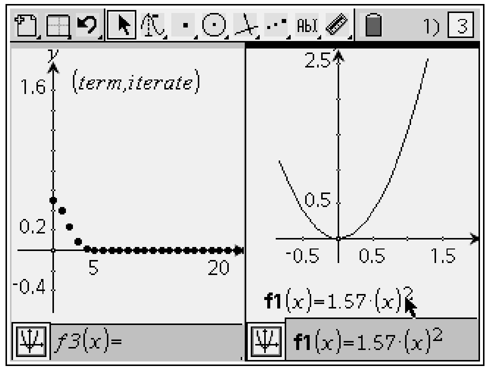
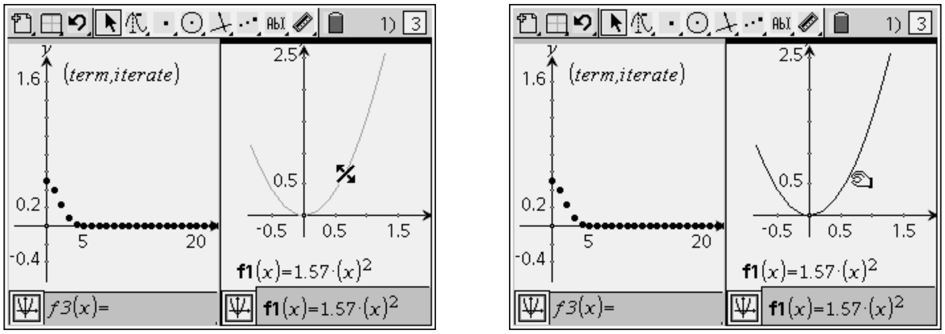
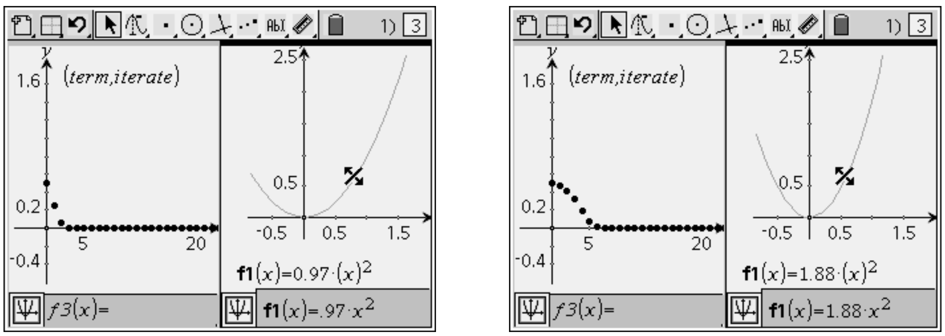


Figure 2: Graph of Sequence and Function

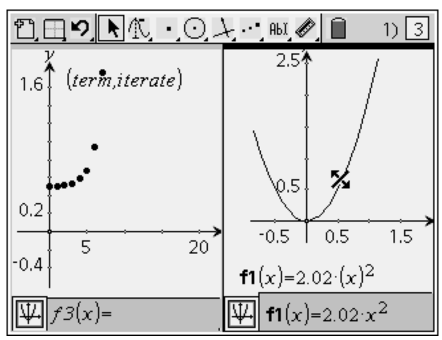
Since TI-Nspire CAS technology allowed them to grab the function and move it (See Figure 3), they did so and watched as the scatter plot changed in real-time (dynamically linked representations). They expected the way in which convergence occurred to change, but they did not expect to see divergence.



Dilation Tool / Grabbing the Parabola



Morphing the Parabola / Slowing Convergence



Surprising Divergence

Figure 3: Dynamically Changing *f* and the Sequence

As a result of their observations, the students decided to try to find the function that formed the boundary between convergence and divergence. As they manipulated the function by grabbing and moving the parabola -- making it wider and then narrower -- they found that the function providing the boundary between convergence and divergence was $f(x)=2x^2$ (See Figure 4).

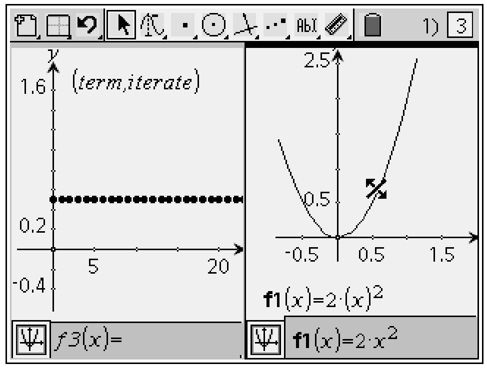


Figure 4: Fixed Point for the Iterated Function

The question that was now raised for the students: Whether or not this same boundary function would hold for any seed value? To test this, the students proceeded to try the seed $x_0 = 0.25$ while leaving the function as $f(x)=2x^2$. The resulting scatter plot quickly answered their question (See Figure 5).

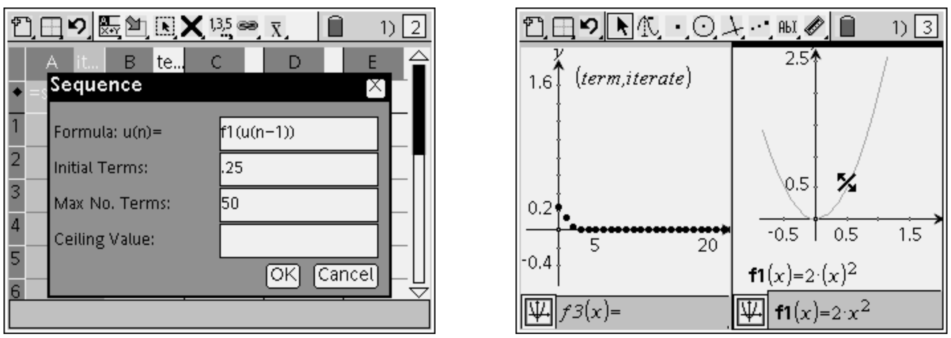
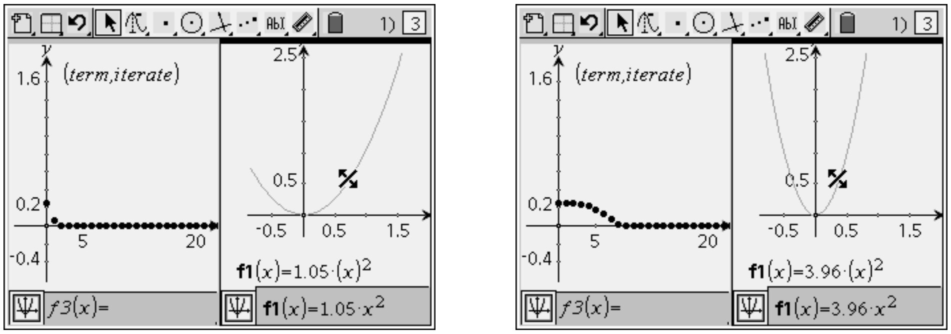
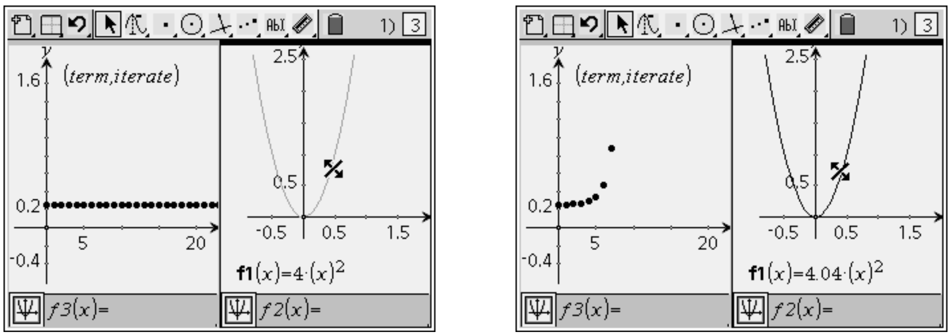


Figure 5: Changing the Sequence Seed Value

As all pages in the problem were automatically updated when the seed value changed, the students saw that the scatter plot now converged to zero. Wanting to find a new boundary function, they manipulated *f* again obtaining the function, $f(x)=4x^2$ (See Figure 6). It was at this point that the students noticed the connection between the parameter *a* in $f(x)=ax^2$ and the seed value, x_0.



Convergence / Slowing Convergence



Fixed Point Boundary / Divergence

Figure 6: Finding the Boundary Function for $x_0 = 0.25$

It was fortunate that the students had chosen $x_0 = 0.25$ since its reciprocal is readily recognized. The students quickly conjectured that the boundary function for convergence given an initial seed value of x_0 would be $f(x)=ax^2$ where $a=\frac{1}{x_0}$. As the students discussed this in their groups, one student used the TI-Nspire CAS handheld and noticed that $f\left(\frac{1}{a}\right)=\frac{1}{a}$ and that this behaved like the identity function.

Dr. Lapp recommends that students and educators take about a week at the beginning of the semester to become familiar with the basic functionality of TI-Nspire[1] technology, and to think about how it might change what they do in the classroom. Classroom educators should become familiar with the available research supporting why dynamically-linked representations are so important and effective as a teaching tool. This will motivate them to realize the utility of TI-Nspire technology's capabilities and support their own curriculum materials in a way which is consistent with the research.

(June, 2007)

[1] In this pre-production version of TI-Nspire CAS technology, Dr. Lapp noted that students took longer to learn to navigate the device than with previous graphing calculators. Also, at this point, slope fields had not yet been added to the feature set of the handheld.